Control System Toolbox

For Use with MATLAB®

- Computation
- Visualization
- Programming

Getting Started

Version 6

The MathWorks

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Getting Started with the Control System Toolbox

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What Is the Control System Toolbox?

Introduction

MATLAB® has a rich collection of functions immediately useful to the control engineer or system theorist. Complex arithmetic, eigenvalues, root-finding, matrix inversion, and fast Fourier transforms are just a few examples of important numerical tools found in MATLAB. More generally, the MATLAB linear algebra, matrix computation, and numerical analysis capabilities provide a reliable foundation for control system engineering as well as many other disciplines.

The Control System Toolbox builds on the foundations of MATLAB to provide functions designed for control engineering. The Control System Toolbox is a collection of algorithms, written mostly as M-files, that implements common control system design, analysis, and modeling techniques. Convenient graphical user interfaces (GUIs) simplify typical control engineering tasks.

Control systems can be modeled as transfer functions, in zero-pole-gain or state-space form, allowing you to use both classical and modern control techniques. You can manipulate both continuous-time and discrete-time systems. Conversions between various model representations are provided. Time responses, frequency responses, and root loci can be computed and graphed. Other functions allow pole placement, optimal control, and estimation. Finally, the Control System Toolbox is open and extensible. You can create custom M-files to suit your particular application.

Installation

Instructions for installing the Control System Toolbox can be found in the MATLAB Installation documentation for your platform. We recommend that you store the files from this toolbox in a subdirectory named `control` under the main `matlab` directory. To determine if the Control System Toolbox is already installed on your system, check for a subdirectory named `control` within the main toolbox directory or folder.

Demos

The Control System Toolbox provides demonstration files that show you how to use the toolbox to perform control design tasks in various settings. To run these demos, type

```
demo
```

at the MATLAB prompt. This opens the **Demos** pane in the Help browser. Select **Toolboxes** and then **Control System** to see a list of available demos. Alternatively, if you have the Help browser open, you can select the **Demos** pane directly and follow the same procedure.

In addition, "Design Case Studies" in the online documentation contains detailed examples of various design problems.

Using the Documentation

If you are a new user, this guide, Getting Started with the Control System Toolbox, is written for you. Specifically, you will learn

- How to build and manipulate linear time-invariant models of dynamical systems
- How to analyze such models and plot their time and frequency responses
- How to design compensators using root locus and pole placement techniques

In addition, this guide discusses model order reduction, linear quadratic Gaussian (LQG) techniques, and presents examples that show how to use these techniques.

This guide is available online under the Control System Toolbox. The rest of the toolbox documentation is also available online; click **Control System Toolbox** to open its product page, which is a roadmap with links to the Control System Toolbox documentation and to PDF versions of the same documentation.

If you are an experienced toolbox user, see the online documentation for detailed discussions of control system design topics, including the following:

- “Release Notes” — Details on the latest release
- “Creating and Manipulating Models” — In-depth information on how to create and manipulate linear models and linear time-invariant (LTI) arrays, which are data objects that you can use to store collections of linear models in one variable
- “Customization” — Information about setting plot properties, including how to set preferences that persist from session to session
- “Design Case Studies” — Worked examples, including Kalman filtering and MIMO design
- “Reliable Computations” — Numerical stability and accuracy issues
- “GUI Reference” — Complete descriptions of the LTI Viewer and SISO Design Tool, which are graphical user interfaces (GUIs) that you can use to analyze systems and design SISO compensators

All toolbox users should use the online Control System Toolbox Function Reference for reference information on functions and tools. For functions, reference descriptions include a synopsis of the function's syntax, as well as a complete explanation of options and operation. Many reference descriptions also include helpful examples, a description of the function's algorithm, and references to additional reading material. For GUI-based tools, the descriptions include options for invoking the tool.

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Building Models

Linear Models

Typically, control engineers begin by developing a mathematical description of the dynamic system that they want to control. The system to be controlled is called a *plant*. As an example of a plant, this section uses the DC motor. This section develops the differential equations that describe the electromechanical properties of a DC motor with an inertial load. It then shows you how to use the Control System Toolbox to build linear models based on these equations.

Linear Model Representations

The Control System Toolbox supports the following model representations:

- State-space models (SS) of the form

  $$\frac{dx}{dt} = Ax + Bu$$
  $$y = Cx + Du$$

  where A, B, C, and D are matrices of appropriate dimensions, x is the state vector, and u and y are the input and output vectors.
- Transfer functions (TF), for example,

  $$H(s) = \frac{s+2}{s^2+s+10}$$

- Zero-pole-gain (ZPK) models, for example,

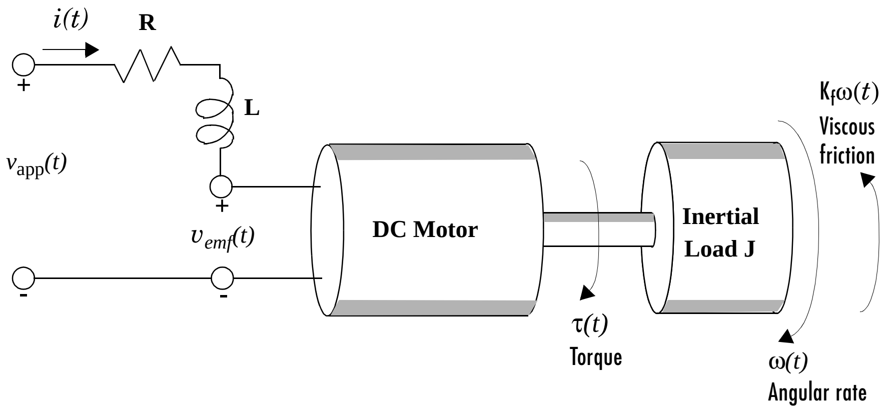
  $$H(z) = 3\frac{(z+1+j)(z+1-j)}{(z+0.2)(z+0.1)}$$

- Frequency response data (FRD) models, which consist of sampled measurements of a system's frequency response. For example, you can store experimentally collected frequency response data in an FRD model.

Note The design of FRD models is a specialized subject that this guide does not address. See Frequency Response Data (FRD) Models under Creating and Manipulating Models in the online documentation for a discussion of this topic.

SISO Example: the DC Motor

A simple model of a DC motor driving an inertial load shows the angular rate of the load, $\omega(t)$, as the output and applied voltage, $v_{app}(t)$, as the input. The ultimate goal of this example is to control the angular rate by varying the applied voltage. This picture shows a simple model of the DC motor.



A Simple Model of a DC Motor Driving an Inertial Load

In this model, the dynamics of the motor itself are idealized; for instance, the magnetic field is assumed to be constant. The resistance of the circuit is denoted by R and the self-inductance of the armature by L. If you are unfamiliar with the basics of DC motor modeling, consult any basic text on physical modeling. The important thing here is that with this simple model and basic laws of physics, it is possible to develop differential equations that describe the behavior of this electromechanical system. In this example, the relationships between electric potential and mechanical force are Faraday's law of induction and Ampère's law for the force on a conductor moving through a magnetic field.

Mathematical Derivation

The torque τ seen at the shaft of the motor is proportional to the current i induced by the applied voltage,

$$\tau(t) = K_m i(t)$$

where K_m, the armature constant, is related to physical properties of the motor, such as magnetic field strength, the number of turns of wire around the conductor coil, and so on. The back (induced) electromotive force, v_{emf}, is a voltage proportional to the angular rate ω seen at the shaft,

$$v_{emf}(t) = K_b \omega(t)$$

where K_b, the emf constant, also depends on certain physical properties of the motor.

The mechanical part of the motor equations is derived using Newton's law, which states that the inertial load J times the derivative of angular rate equals the sum of all the torques about the motor shaft. The result is this equation,

$$J\frac{d\omega}{dt} = \sum \tau_i = -K_f \omega(t) + K_m i(t)$$

where $K_f \omega$ is a linear approximation for viscous friction.

Finally, the electrical part of the motor equations can be described by

$$v_{app}(t) - v_{emf}(t) = L\frac{di}{dt} + Ri(t)$$

or, solving for the applied voltage and substituting for the back emf,

$$v_{app}(t) = L\frac{di}{dt} + Ri(t) + K_b \omega(t)$$

This sequence of equations leads to a set of two differential equations that describe the behavior of the motor, the first for the induced current,

$$\frac{di}{dt} = -\frac{R}{L}i(t) - \frac{K_b}{L}\omega(t) + \frac{1}{L}v_{app}(t)$$

and the second for the resulting angular rate,

$$\frac{d\omega}{dt} = -\frac{1}{J}K_f \omega(t) + \frac{1}{J}K_m i(t)$$

State-Space Equations for the DC Motor

Given the two differential equations derived in the last section, you can now develop a state-space representation of the DC motor as a dynamic system. The current i and the angular rate ω are the two states of the system. The applied voltage, v_{app}, is the input to the system, and the angular velocity ω is the output.

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} i \\ \omega \end{bmatrix} + [0] \cdot v_{app}(t)$$

State-Space Representation of the DC Motor Example

Constructing SISO Models

Once you have a set of differential equations that describe your plant, you can construct SISO models using simple commands in the Control System Toolbox. The following sections discuss

- Constructing a state-space model of the DC motor
- Converting between model representations
- Creating transfer function and zero/pole/gain models

Constructing a State-Space Model of the DC Motor

Listed below are nominal values for the various parameters of a DC motor.

```
R= 2.0 % Ohms
L= 0.5 % Henrys
Km = .015 % torque constant
Kb = .015 % emf constant
Kf = 0.2 % Nms
J= 0.02 % kg.m^2
```

Given these values, you can construct the numerical state-space representation using the ss function.

```
A = [-R/L -Kb/L; Km/J -Kf/J]
B = [1/L; 0];
C = [0 1];
D = [0];
sys_dc = ss(A,B,C,D)
```

This is the output of the last command.

```
a = 
                       x1         x2
           x1          -4      -0.03
           x2        0.75        -10


b = 
                       u1
           x1           2
           x2           0


c = 
                       x1         x2
           y1           0          1


d = 
                       u1
           y1           0
```

Converting Between Model Representations

Now that you have a state-space representation of the DC motor, you can convert to other model representations, including transfer function (TF) and zero/pole/gain (ZPK) models.

Transfer Function Representation. You can use `tf` to convert from the state-space representation to the transfer function. For example, use this code to convert to the transfer function representation of the DC motor.

```
sys_tf = tf(sys_dc)

Transfer function:
       1.5
------------------
s^2 + 14 s + 40.02
```

Zero/Pole/Gain Representation. Similarly, the `zpk` function converts from state-space or transfer function representations to the zero/pole/gain format. Use this code to convert from the state-space representation to the zero/pole/gain form for the DC motor.

```
sys_zpk = zpk(sys_dc)

Zero/pole/gain:
        1.5
-------------------
(s+4.004) (s+9.996)
```

Note The state-space representation is best suited for numerical computations. For highest accuracy, convert to state space prior to combining models and avoid the transfer function and zero/pole/gain representations, except for model specification and inspection. See "Reliable Computations" in the online Control System Toolbox documentation for more information on numerical issues.

Constructing Transfer Function and Zero/Pole/Gain Models

In the DC motor example, the state-space approach produces a set of matrices that represents the model. If you choose a different approach, you can construct the corresponding models using `tf`, `zpk`, `ss`, or `frd`.

```
sys = tf(num,den)              % Transfer function
sys = zpk(z,p,k)               % Zero/pole/gain
sys = ss(a,b,c,d)              % State-space
sys = frd(response,frequencies) % Frequency response data
```

For example, if you want to create the transfer function of the DC motor directly, use these commands.

```
s = tf('s');
sys_tf = 1.5/(s^2+14*s+40.02)
```

The Control System Toolbox builds this transfer function.

```
Transfer function:
        1.5
--------------------
s^2 + 14 s + 40.02
```

Alternatively, you can create the transfer function by specifying the numerator and denominator with this code.

```
sys_tf = tf(1.5,[1 14 40.02])

Transfer function:
       1.5
------------------
s^2 + 14 s + 40.02
```

To build the zero/pole/gain model, use this command.

```
sys_zpk = zpk([],[-9.996 -4.004], 1.5)
```

This is the resulting zero/pole/gain representation.

```
Zero/pole/gain:
        1.5
-------------------
(s+9.996) (s+4.004)
```

Discrete Time Systems

The Control System Toolbox provides full support for discrete-time systems. You can create discrete systems in the same way that you create analog systems; the only difference is that you must specify a sample time period for any model you build. For example,

```
sys_disc = tf(1, [1 1], .01);
```

creates a SISO model in the transfer function format.

```
Transfer function:
  1
-----
z + 1

Sampling time: 0.01
```

Adding Time Delays to Discrete-Time Models

You can add time delays to discrete-time models by specifying an input or output time delay when building the model. The time delay must be a nonnegative integer that represents a multiple of the sampling time. For example,

```
sys_delay = tf(1, [1 1], 0.01,'outputdelay',5);
```

produces a system with an output delay of 0.05 second.

```
Transfer function:
           1
z^(-5) * -----
         z + 1

Sampling time: 0.01
```

Adding Delays to Linear Models

You can add time delays to linear models by specifying an input or output delay when building a model. For example, to add an input delay to the DC motor, use this code.

```
sys_tfdelay = tf(1.5, [1 14 40.02],'inputdelay',0.05)
```

The Control System Toolbox constructs the DC motor transfer function, but adds a 0.05 second delay.

```
Transfer function:
                          1.5
exp(-0.05*s) * ------------------
               s^2 + 14 s + 40.02
```

For a complete description of adding time delays to models, see “Time Delays” online under “Creating and Manipulating Models.”

LTI Objects

For convenience, the Control System Toolbox uses custom data structures called *LTI objects* to store model-related data. For example, the variable `sys_dc` created for the DC motor example is called an *SS object*. There are also TF, ZPK, and FRD objects for transfer function, zero/pole/gain, and frequency data response models respectively. The four LTI objects encapsulate the model data and enable you to manipulate linear systems as single entities rather than as collections of vectors or matrices.

To see what LTI objects contain, use the `get` command. This code describes the contents of `sys_dc` from the DC motor example.

```
get(sys_dc)
             a: [2x2 double]
             b: [2x1 double]
             c: [0 1]
             d: 0
             e: []
     StateName: {2x1 cell}
            Ts: 0
       ioDelay: 0
    InputDelay: 0
   OutputDelay: 0
```

```
         InputName: {''}
        OutputName: {''}
        InputGroup: {0x2 cell}
       OutputGroup: {0x2 cell}
             Notes: {}
          UserData: []
```

You can manipulate the data contained in LTI objects using the `set` command; see the Control System Toolbox online reference pages for descriptions of `set` and `get`.

Another convenient way to set or retrieve LTI model properties is to access them directly using dot notation. For example, if you want to access the value of the `A` matrix, instead of using `get`, you can type

```
sys_dc.a
```

at the MATLAB prompt. MATLAB returns the `A` matrix.

```
ans =

   -4.0000   -0.0300
    0.7500  -10.0000
```

Similarly, if you want to change the values of the `A` matrix, you can do so directly, as this code shows.

```
A_new = [-4.5 -0.05; 0.8 -12.0];
sys_dc.a = A_new;
```

For more information on LTI properties, type `ltiprops` at the MATLAB prompt. For a complete description of LTI objects, see “Creating and Manipulating Models” online under the Control System Toolbox.

MIMO Models

You can use the same functions that apply to SISO systems to create multiple-input, multiple-output (MIMO) models, including arbitrary MIMO transfer functions and zero/pole/gain models. This section begins with an example of how to build a MIMO system. It then discusses how to build MIMO transfer functions by concatenating SISO transfer functions and how to access and manipulate individual SISO transfer functions contained in a MIMO model.

MIMO Example: Jet Transport Aircraft

This example shows how to build a MIMO model of a jet transport. Since the development of a physical model for a jet aircraft is lengthy, only the state-space equations are presented here. See any standard text in aviation for a more complete discussion of the physics behind aircraft flight.

The jet model during cruise flight at MACH = 0.8 and H = 40,000 ft. is

```
A = [-0.0558   -0.9968    0.0802    0.0415
      0.5980   -0.1150   -0.0318         0
     -3.0500    0.3880   -0.4650         0
           0    0.0805    1.0000         0];

B = [ 0.0073         0
     -0.4750    0.0077
      0.1530    0.1430
           0         0];

C = [0     1     0     0
     0     0     0     1];

D = [0     0
     0     0];
```

Use the following commands to specify this state-space model as an LTI object and attach names to the states, inputs, and outputs.

```
states = {'beta' 'yaw' 'roll' 'phi'};
inputs = {'rudder' 'aileron'};
outputs = {'yaw rate' 'bank angle'};
```

```
sys_mimo = ss(A,B,C,D,'statename',states,...
                  'inputname',inputs,...
                  'outputname',outputs);
```

You can display the LTI model by typing `sys_mimo`.

```
sys_mimo

a =
                        beta          yaw         roll          phi
          beta       -0.0558      -0.9968       0.0802       0.0415
           yaw         0.598       -0.115      -0.0318            0
          roll         -3.05        0.388       -0.465            0
           phi             0       0.0805            1            0


b =
                      rudder      aileron
          beta        0.0073            0
           yaw        -0.475       0.0077
          roll         0.153        0.143
           phi             0            0


c =
                        beta          yaw         roll          phi
      yaw rate             0            1            0            0
    bank angle             0            0            0            1


d =
                      rudder      aileron
      yaw rate             0            0
    bank angle             0            0

Continuous-time model.
```

The model has two inputs and two outputs. The units are radians for `beta` (sideslip angle) and `phi` (bank angle) and radians/sec for `yaw` (yaw rate) and `roll` (roll rate). The rudder and aileron deflections are in degrees.

As in the SISO case, use `tf` to derive the transfer function representation.

```
tf(sys_mimo)

Transfer function from input "rudder" to output...
               -0.475 s^3 - 0.2479 s^2 - 0.1187 s - 0.05633
 yaw rate:  ---------------------------------------------------
            s^4 + 0.6358 s^3 + 0.9389 s^2 + 0.5116 s + 0.003674

                         0.1148 s^2 - 0.2004 s - 1.373
 bank angle:  ---------------------------------------------------
              s^4 + 0.6358 s^3 + 0.9389 s^2 + 0.5116 s + 0.003674

Transfer function from input "aileron" to output...
            0.0077 s^3 - 0.0005372 s^2 + 0.008688 s + 0.004523
 yaw rate:  ---------------------------------------------------
            s^4 + 0.6358 s^3 + 0.9389 s^2 + 0.5116 s + 0.003674

                        0.1436 s^2 + 0.02737 s + 0.1104
 bank angle:  ---------------------------------------------------
              s^4 + 0.6358 s^3 + 0.9389 s^2 + 0.5116 s + 0.003674
```

Constructing MIMO Transfer Functions

MIMO transfer functions are two-dimensional arrays of elementary SISO transfer functions. There are two ways to specify MIMO transfer function models:

- Concatenation of SISO transfer function models
- Using `tf` with cell array arguments

Concatenation of SISO Models

Consider the following single-input, two-output transfer function.

$$H(s) = \begin{bmatrix} \frac{s-1}{s+1} \\ \frac{s+2}{s^2+4s+5} \end{bmatrix}$$

You can specify $H(s)$ by concatenation of its SISO entries. For instance,

```
h11 = tf([1 -1],[1 1]);
h21 = tf([1 2],[1 4 5]);
```

or, equivalently,

```
s = tf('s')
h11 = (s-1)/(s+1);
h21 = (s+2)/(s^2+4*s+5);
```

can be concatenated to form $H(s)$.

```
H = [h11; h21]
```

This syntax mimics standard matrix concatenation and tends to be easier and more readable for MIMO systems with many inputs and/or outputs. See "Model Interconnection Functions" in "Creating and Manipulating Models" online for more details on concatenation operations for LTI systems.

Using the tf Function with Cell Arrays

Alternatively, to define MIMO transfer functions using `tf`, you need two cell arrays (say, `N` and `D`) to represent the sets of numerator and denominator polynomials, respectively. See "Structures and Cell Arrays" in the MATLAB online documentation for more details on cell arrays.

For example, for the rational transfer matrix $H(s)$, the two cell arrays `N` and `D` should contain the row-vector representations of the polynomial entries of

$$N(s) = \begin{bmatrix} s-1 \\ s+2 \end{bmatrix} \qquad D(s) = \begin{bmatrix} s+1 \\ s^2+4s+5 \end{bmatrix}$$

You can specify this MIMO transfer matrix $H(s)$ by typing

```
N = {[1 -1];[1 2]};   % Cell array for N(s)
D = {[1 1];[1 4 5]}; % Cell array for D(s)
H = tf(N,D)
```

The Control System Toolbox responds with

```
Transfer function from input to output...
      s - 1
 #1:  -----
      s + 1
```

```
          s + 2
 #2:  -------------
      s^2 + 4 s + 5
```

Notice that both N and D have the same dimensions as H. For a general MIMO transfer matrix $H(s)$, the cell array entries N{i,j} and D{i,j} should be row-vector representations of the numerator and denominator of $H_{ij}(s)$, the ijth entry of the transfer matrix $H(s)$.

Accessing I/O Pairs in MIMO Systems

Once you have defined a MIMO system, you can access and manipulate I/O pairs by specifying input and output pairs of the system. For instance, if sys_mimo is a MIMO system with two inputs and three outputs,

```
sys_mimo(3,1)
```

extracts the subsystem, mapping the first input to the third output. Row indices select the outputs and column indices select the inputs. Similarly,

```
sys_mimo(3,1) = tf(1,[1 0])
```

redefines the transfer function between the first input and third output as an integrator.

Arrays of Linear Models

You can specify and manipulate collections of linear models as single entities using LTI arrays. For example, if you want to vary the Kb and Km parameters for the DC motor and store the resulting state-space models, use this code.

```
K = [0.1 0.15 0.2]; % Several values for Km and Kb
A1 = [-R/L -K(1)/L; K(1)/J -Kf/J];
A2 = [-R/L -K(2)/L; K(2)/J -Kf/J];
A3 = [-R/L -K(3)/L; K(3)/J -Kf/J];
sys_lti(:,:,1)= ss(A1,B,C,D);
sys_lti(:,:,2)= ss(A2,B,C,D);
sys_lti(:,:,3)= ss(A3,B,C,D);
```

(Note that Kb and Km must be equal, so K represents both parameters in the state-space equations.) The number of inputs and outputs must be the same for all linear models encapsulated by the LTI array, but the model order (number of states) can vary from model to model within a single LTI array.

The LTI array sys_lti contains the state-space models for each value of K. Type sys_lti to see the contents of the LTI array.

```
Model sys_lti(:,:,1,1)
======================


  a =
                     x1           x2
           x1        -4         -0.2
           x2         5          -10
.
.
.
Model sys_lti(:,:,2,1)
======================


  a =
                     x1           x2
           x1        -4         -0.3
           x2       7.5          -10
.
.
```

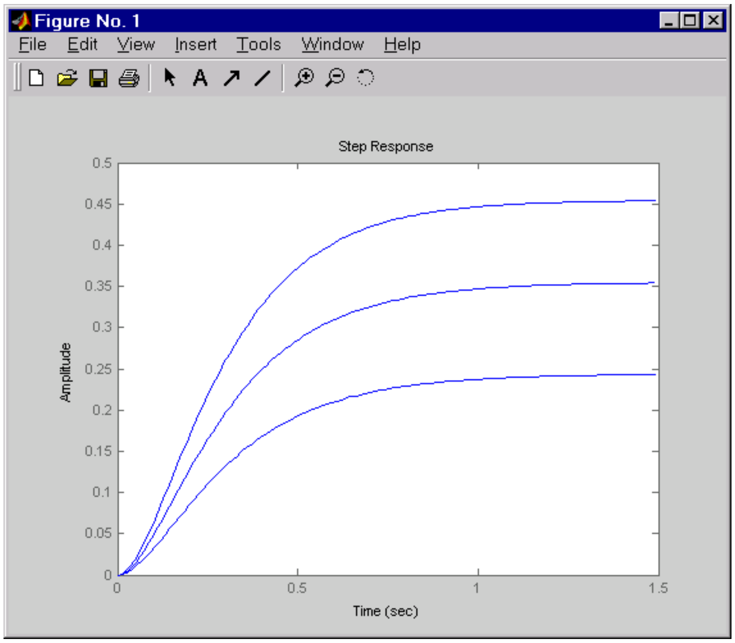
```
.
Model sys_lti(:,:,3,1)
======================

  a = 
                     x1           x2
           x1        -4         -0.4
           x2        10          -10
.
.
.
3x1 array of continuous-time state-space models.
```

You can manipulate the LTI array like any other object in the Control System Toolbox. For example,

```
step(sys_lti)
```

produces a plot containing step responses for all three state-space models.



Step Responses for an LTI Array Containing Three Models

LTI arrays are useful for performing batch analysis on an entire set of models. For more information, see “Arrays of LTI Models” online under “Creating and Manipulating Models” in the Control System Toolbox.

Model Characteristics

The Control System Toolbox contains commands to query model characteristics such as the I/O dimensions, poles, zeros, and DC gain. These commands apply to both continuous- and discrete-time models. Their LTI-based syntax is summarized in the table below.

Commands to Query Model Characteristics

Command	Description
`size(model_name)`	Number of inputs and outputs
`ndims(model_name)`	Number of dimensions
`isct(model_name)`	Returns 1 for continuous systems
`isdt(model_name)`	Returns 1 for discrete systems
`hasdelay(model_name)`	True if system has delays
`pole(model_name)`	System poles
`zero(model_name)`	System (transmission) zeros
`dcgain(model_name)`	DC gain
`norm(model_name)`	System norms (H_2 and L_∞)
`covar(model_name,W)`	Covariance of response to white noise

Interconnecting Linear Models

You can perform simple operations on LTI models, such as addition, multiplication, or concatenation. Addition performs a parallel interconnection. For example, typing

```
tf(1,[1 0]) + tf([1 1],[1 2])   % 1/s + (s+1)/(s+2)
```

produces this transfer function.

```
Transfer function:
s^2 + 2 s + 2
-------------
  s^2 + 2 s
```

Multiplication performs a series interconnection. For example, typing

```
2 * tf(1,[1 0])*tf([1 1],[1 2])   % 2*1/s*(s+1)/(s+2)
```

produces this cascaded transfer function.

```
Transfer function:
2 s + 2
---------
s^2 + 2 s
```

If the operands are models of different types, the resulting model type is determined by precedence rules; see “Precedence Rules” under “Creating and Manipulating Models” online for more information. State-space models have the highest precedence while transfer functions have the lowest precedence. Hence the sum of a transfer function and a state-space model is always a state-space model.

Other available operations include system inversion, transposition, and pertransposition; see “Arithmetic Operations” online under “Creating and Manipulating Models.” The Control System Toolbox also supports matrix-like indexing for extracting subsystems; see “Extracting and Modifying Subsystems” online in “Operations on LTI Models” (also under “Creating and Manipulating Models”) for more information.

You can also use the `series` and `parallel` functions as substitutes for multiplication and addition, respectively.

Equivalent Ways to Interconnect Systems

Operator	Function	Resulting Transfer Function
`sys1 + sys2`	`parallel(sys1,sys2)`	Systems in parallel
`sys1 * sys2`	`series(sys2,sys1)`	Cascaded systems

Feedback Interconnection

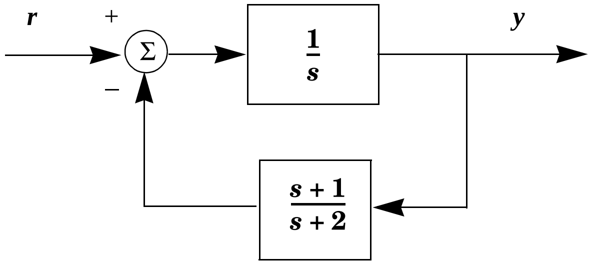
You can use the `feedback` and `lft` functions to derive closed-loop models. For example,

```
sys_f = feedback(tf(1,[1 0]), tf([1 1],[1 2]))
```

computes the closed-loop transfer function from *r* to *y* for the feedback loop shown below. The result is

```
Transfer function:
    s + 2
-------------
s^2 + 3 s + 1
```

This figure shows the interconnected system in block diagram format.



Feedback Interconnection

You can use the `lft` function to create more complicated feedback structures. This function constructs the linear fractional transformation of two systems. See the online reference page for more information.

Continuous/Discrete Conversions

The commands c2d, d2c, and d2d perform continuous to discrete, discrete to continuous, and discrete to discrete (resampling) conversions, respectively.

```
sysd = c2d(sysc,Ts)  % Discretization w/ sample period Ts
sysc = d2c(sysd)     % Equivalent continuous-time model
sysd1= d2d(sysd,Ts)  % Resampling at the period Ts
```

Various discretization/interpolation methods are available, including zero-order hold (default), first-order hold, Tustin approximation with or without prewarping, and matched zero-pole. For example,

```
sysd = c2d(sysc,Ts,'foh')   % Uses first-order hold
sysc = d2c(sysd,'tustin')   % Uses Tustin approximation
```

Discrete DC Motor Model

You can digitize the DC motor plant using the c2d function and selecting an appropriate sample time. Choosing the right sample time involves many factors, including the performance you want to achieve, the fastest time constant in your system, and the speed at which you expect your controller to run. For this example, choose a time constant of 0.01 second. See “SISO Example: the DC Motor” on page 2-3 for the construction of the SS object sys_dc.

```
Ts=0.01;
sysd=c2d(sys_dc,Ts)

a =
                       x1           x2
           x1     0.96079  -0.00027976
           x2    0.006994      0.90484


b =
                       u1
           x1    0.019605
           x2  7.1595e-005
```

```
c =
                        x1           x2
           y1            0            1


d =
                        u1
           y1            0

Sampling time: 0.01
Discrete-time model.
```

To see the discrete-time transfer function for the digital DC motor, use `tf` to convert the model.

```
fd=tf(sysd)

Transfer function:
7.16e-005 z + 6.833e-005
------------------------
 z^2 - 1.866 z + 0.8694

Sampling time: 0.01
```

Model Order Reduction

You can derive reduced-order models with the following commands.

Model Order Reduction	
`balreal`	Input/output balancing
`minreal`	Minimal realization (pole/zero cancellation)
`modred`	State deletion in I/O balanced realization
`sminreal`	Structurally minimal realization

Use `minreal` to delete uncontrollable or unobservable state dynamics in state-space models, or to cancel pole/zero pairs in transfer functions or zero-pole-gain models. Use `sminreal` to remove any states that are structurally decoupled from the inputs or outputs. For already minimal models, you can further reduce the model order using a combination of `balreal` and `modred`.

Example: Gasifier Model

This example presents a model of a gasifier, a device that converts solid materials into gases. The original model is nonlinear. To load a linearized version of the model, type

```
load ltiexamples
```

at the MATLAB prompt; the gasifier example is stored in the variable named `gasf`. If you type

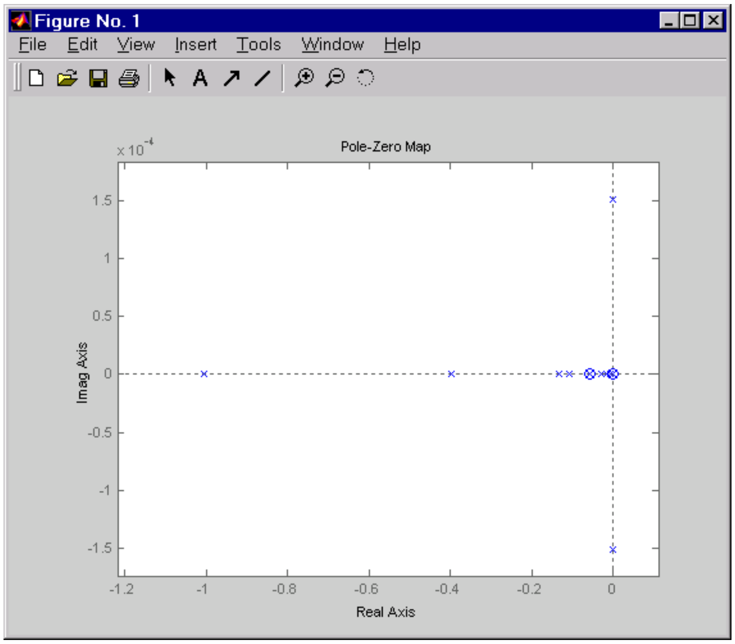
```
size(gasf)
```

MATLAB responds with

```
State-space model with 4 outputs, 6 inputs, and 25 states.
```

Before attempting model order reduction, inspect the pole and zero locations by using `pzmap(gasf)` and then zooming in near the origin. If you don't know how to use the zoom feature on plots, see "Zooming" on page 4-31 for an example.

This figure shows the results.



Pole-Zero Map of the Gasifier Model (Zoomed In)

Since the model displays near pole-zero cancellations, it is a good candidate for model reduction.

SISO Model Order Reduction

As an illustration of the model order reduction tools, this example focuses on a single input/output pair of the gasifier, input 5 to output 3.

```
sys35 = gasf(3,5);
```

To enhance the numerical stability, first scale the system realization with `ssbal`.

```
sys1 = ssbal(sys35);
```

Then use `minreal` to eliminate uncontrollable or unobservable states.

```
sys1 = minreal(sys1);
```

```
size(sys1)
```

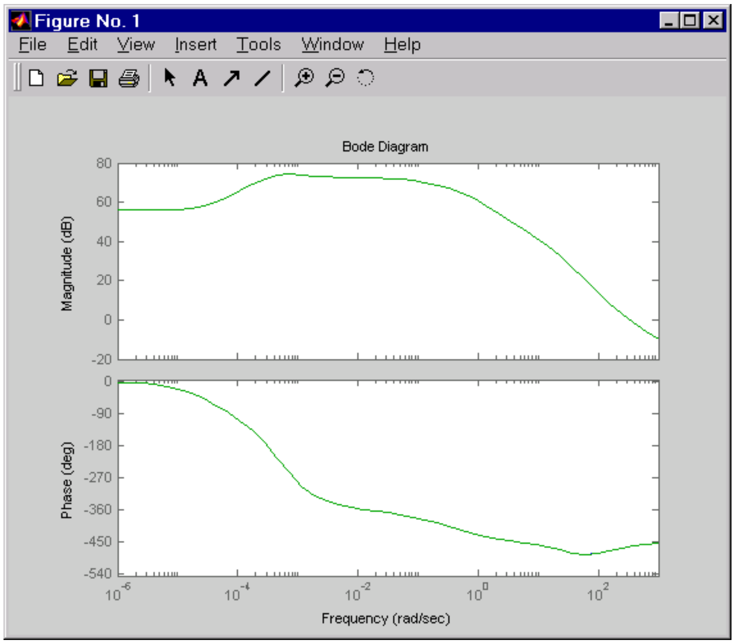
MATLAB responds with

```
State-space model with 1 output, 1 input, and 15 states.
```

The result is a 15th order system. Use this command

```
bode(sys35,sys1);
```

to compare the Bode magnitude and phase of the 25th order model to the reduced-order model. This figure shows the result.



Comparison of Full 25-State Model to the 15-State Reduced Order Models

As the figure shows, there is very little difference in the responses.

Finally, try eliminating states that are weakly affecting the I/O map by using the `balreal` and `modred` functions. First, attempt a balanced realization.

```
[sys1,G] = balreal(sys1);
```

Use `format short e` to view the Hankel singular values stored in variable `G`.

```
G =

   4.5468e+003
   2.6009e+003
   1.8601e+003
   2.5140e+002
   1.5081e+002
   1.1993e+001
   1.1524e+001
   1.0940e+001
   2.8766e+000
   1.3706e+000
   3.5426e-001
   2.2556e-002
   1.2496e-002
   1.0725e-002
   6.2703e-005
```

Small Hankel singular values indicate that the associated states are weakly coupled. You can try discarding the last five states (associated with the five smallest Hankel singular values).

```
sys2 = modred(sys1,11:15);       % Down to 10 states
size(sys2)
```

MATLAB responds with

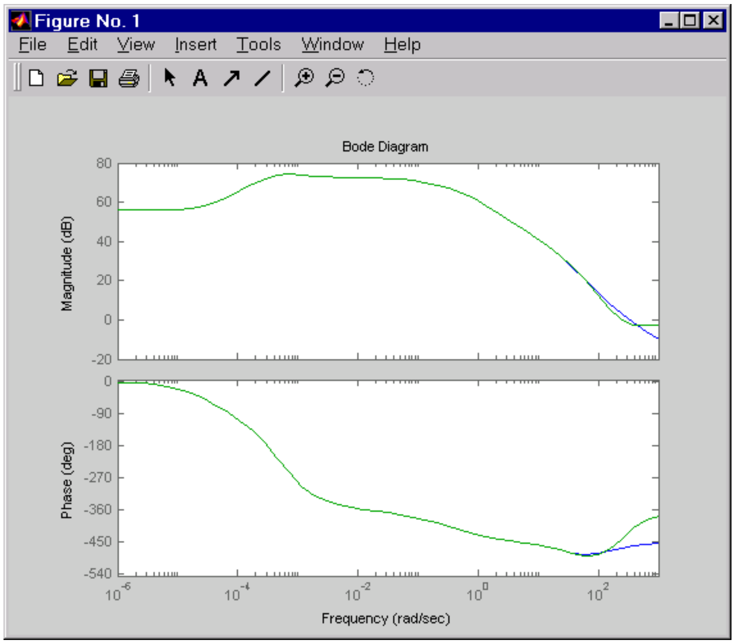
```
State-space model with 1 output, 1 input, and 10 states.
```

Type

```
bode(sys35,sys2);
```

to compare the magnitude and phase of the 10th order model to the 25th order model.

This figure shows the result.



Comparison of 25- and 10-State Models

The figure shows good agreement until the frequency reaches 5 rad/s. This may be acceptable since gasifiers are often low bandwidth systems, and since the models still agree at smaller frequencies. Try experimenting with discarding more Hankel values. With each further reduction, the match to the 25th order model will continue to degrade.

MIMO Model Order Reduction

You can choose not to restrict yourself to individual input/output pairs. The following code does model order reduction on the full MIMO gasifier model.

```
sys1 = ssbal(gasf)      % Scaling

% Compute the minimal realization and balance the model
sys2 = minreal(sys1);    % Down to 17 states
[sys3,G] = balreal(sys2);
```

```
% Discard smallest entry of G by using modred
sys3 = modred(sys3,17);    % Down to 16 states
```

After you get to 16 states, the reduced-order MIMO model begins to deteriorate when compared to the full 25-state MIMO model. Try reducing the model further to see which channels suffer the most degradation.

Acknowledgment

The MathWorks would like to thank ALSTOM Power UK for kindly permitting us to use their gasifier model for this example. This model was issued as part of the ALSTOM Benchmark Challenge on Gasifier Control. For more details see Dixon, R., (1999), “Advanced Gasifier Control,” *Computing & Control Engineering Journal*, IEE, Vol. 10, No. 3, pp. 92-96.

Analyzing Models

LTI Viewer

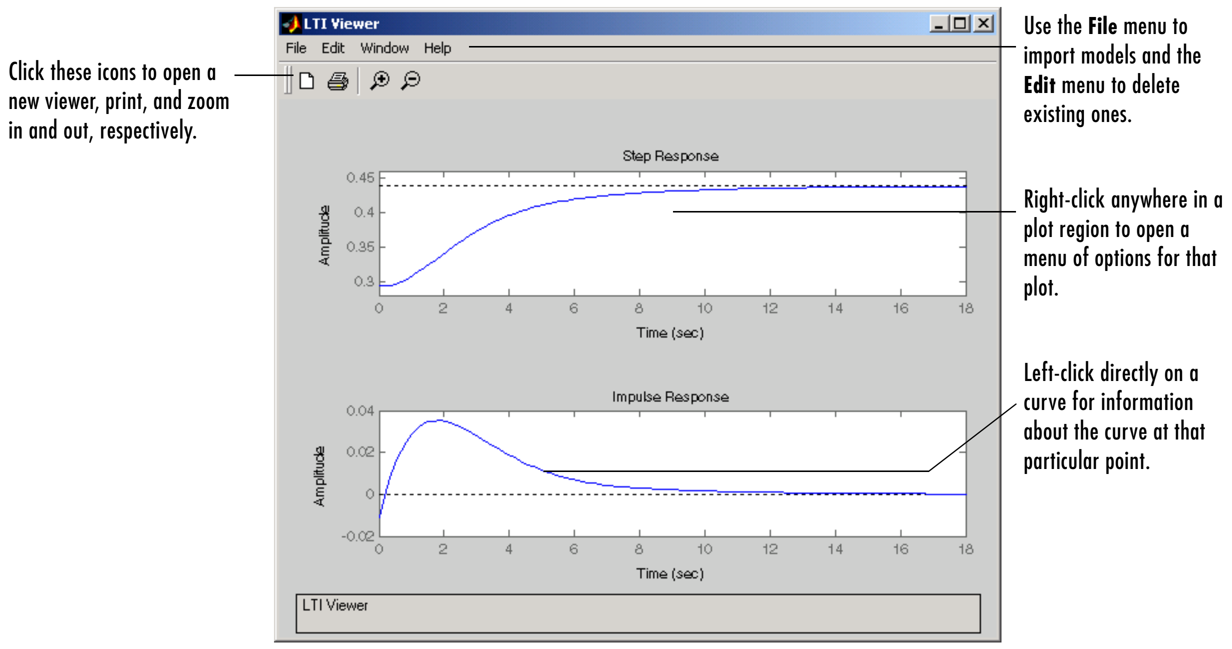
The LTI Viewer is a GUI for viewing and manipulating the response plots of linear models. You can display the following plot types for linear models using the LTI Viewer:

- Step and impulse responses
- Bode and Nyquist plots
- Nichols plots
- Singular values of the frequency response
- Pole/zero plots
- Response to a general input signal
- Unforced response starting from given initial states (only for state-space models)

Note that time responses and pole/zero plots are available only for transfer function, state-space, and zero/pole/gain models.

Note The LTI Viewer displays up to six different plot types simultaneously. You can also analyze the response plots of several linear models at once.

This figure shows an LTI Viewer with two response plots.


Click these icons to open a new viewer, print, and zoom in and out, respectively.
LTI Viewer
File Edit Window Help
Use the **File** menu to import models and the **Edit** menu to delete existing ones.
Step Response
Amplitude
Time (sec)
Right-click anywhere in a plot region to open a menu of options for that plot.
Impulse Response
Left-click directly on a curve for information about the curve at that particular point.
LTI Viewer

The LTI Viewer with Step and Impulse Response Plots

The next section presents an example that shows you how to import a system into the LTI Viewer and how to customize the viewer to fit your requirements.

Example: Time and Frequency Responses of the DC Motor

The section titled “SISO Example: the DC Motor” on page 2-3 presents a DC motor example. If you have not yet built that example, type

```
load ltiexamples
```

at the MATLAB prompt. This loads several LTI models, including a state-space representation of the DC motor called `sys_dc`.

Opening the LTI Viewer

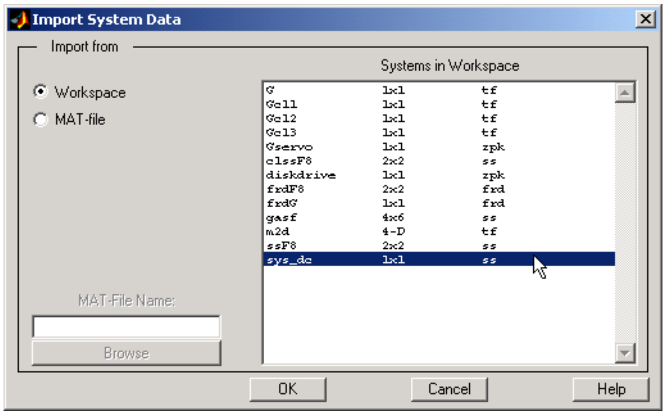
To open the LTI Viewer, type

```
ltiview
```

This opens an LTI Viewer with an empty step response plot window by default.

Importing Models into the LTI Viewer

To import the DC motor model, select **Import** under the **File** menu. This opens the **Import System Data** window, which lists all the models available in your MATLAB workspace.



Systems in Workspace

```
G           1x1    tf
Gcl1        1x1    tf
Gcl2        1x1    tf
Gcl3        1x1    tf
Gservo      1x1    zpk
clssF8      2x2    ss
diskdrive   1x1    zpk
frdF8       2x2    frd
frdG        1x1    frd
gasf        4x6    ss
m2d         4-D    tf
ssF8        2x2    ss
sys_dc      1x1    ss
```

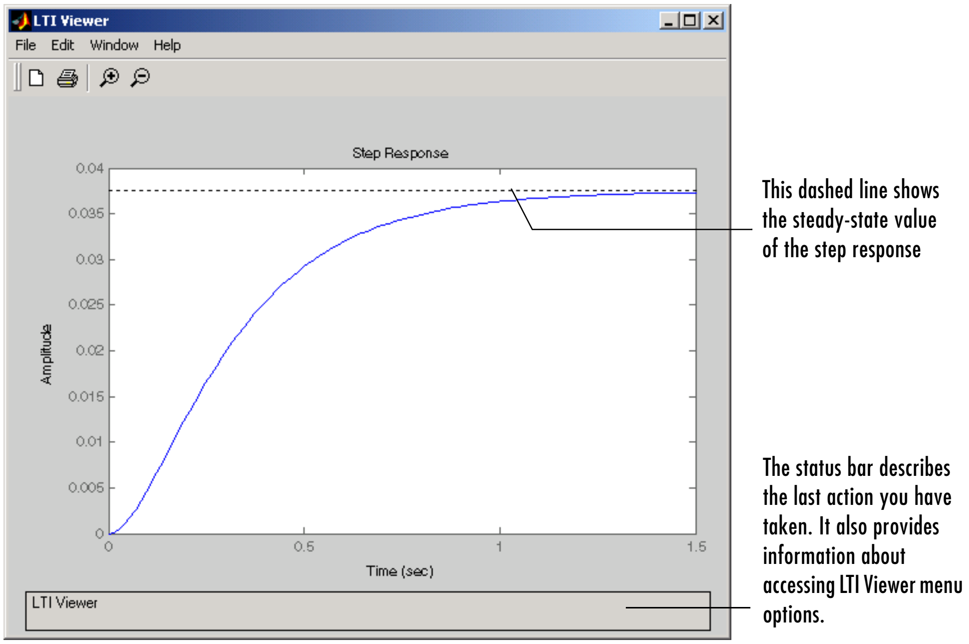
The Import System Data Window with the DC Motor Model Selected

Select sys_dc from the list of available models and click **OK** to close the browser. This imports the DC motor model into the LTI Viewer.

To select more than one model at a time, do the following:

- To select individual (noncontiguous) models, select one model and hold down the **Ctrl** key while selecting additional models. To clear any models, hold down the **Ctrl** key while you click the highlighted model names.
- To select a list of contiguous models, select the first model and hold down the **Shift** key while selecting the last model you want in the list.

The figure below shows the LTI Viewer with a step response for the DC motor example.


LTI Viewer
File Edit Window Help
Step Response
Amplitude
Time (sec)
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
0.5
1
1.5
LTI Viewer
This dashed line shows the steady-state value of the step response
The status bar describes the last action you have taken. It also provides information about accessing LTI Viewer menu options.

Step Response for the DC Motor Example in the LTI Viewer

Alternatively, you can open the LTI Viewer and import the DC motor example directly from the MATLAB prompt.

```
ltiview('step', sys_dc)
```

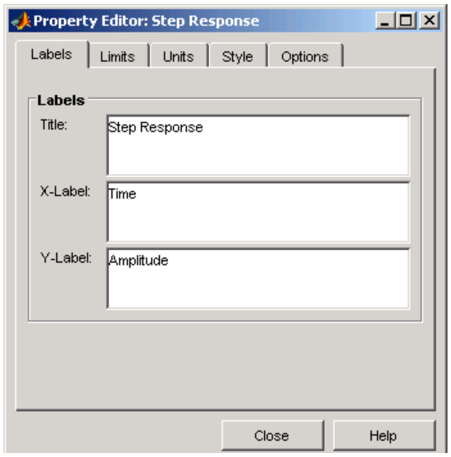
See the `ltiview` reference page for a complete list of options.

Right-Click Menus

The LTI Viewer provides a set of controls and options that you can access by right-clicking your mouse. Once you have imported a model into the LTI Viewer, the options you can select include

- **Plot Type —** Change the plot type. Available types include step, impulse, Bode, Bode magnitude, Nichols, Nyquist, and singular values plots.

- **Systems** — Select or clear any models that you included when you created the response plot.
- **Characteristics** — Add information about the plot. The characteristics available change from plot to plot. For example, Bode plots have stability margins available, but step responses have rise time and steady-state values available.
- **Grid** — Add grids to your plots.
- **Normalize** — Scale responses to fit the view (only available for time-domain plot types).
- **Full View** — Use automatic limits to make the entire curve visible.
- **Properties** — Open the Property Editor.



Labels

Title: Step Response

X-Label: Time

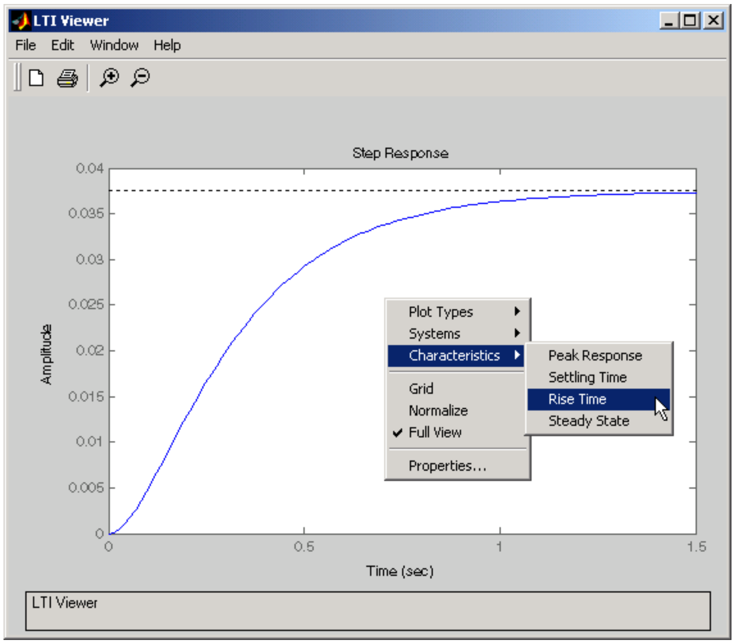
Y-Label: Amplitude

You can use this editor to customize various attributes of your plot. See "Customizing Plot Properties and Preferences" online under "Customization" for a full description of the Property Editor.

Alternatively, you can open the Property Editor by double-clicking in an empty region of the response plot.

Displaying Response Characteristics on a Plot

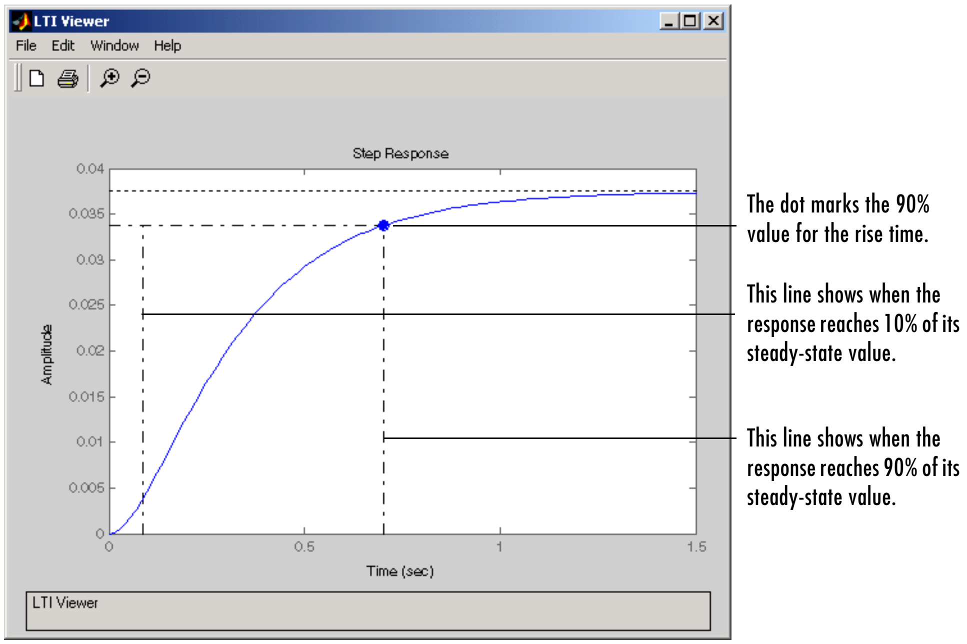
For example, to see the rise time for the DC motor step response, right-click your mouse and select **Rise Time** under **Characteristics**, as this figure illustrates.



Using Right-Click Menus to Display the Rise Time for a Step Response

The rise time is defined as the amount of time it takes the step response to go from 10% to 90% of the steady-state value.

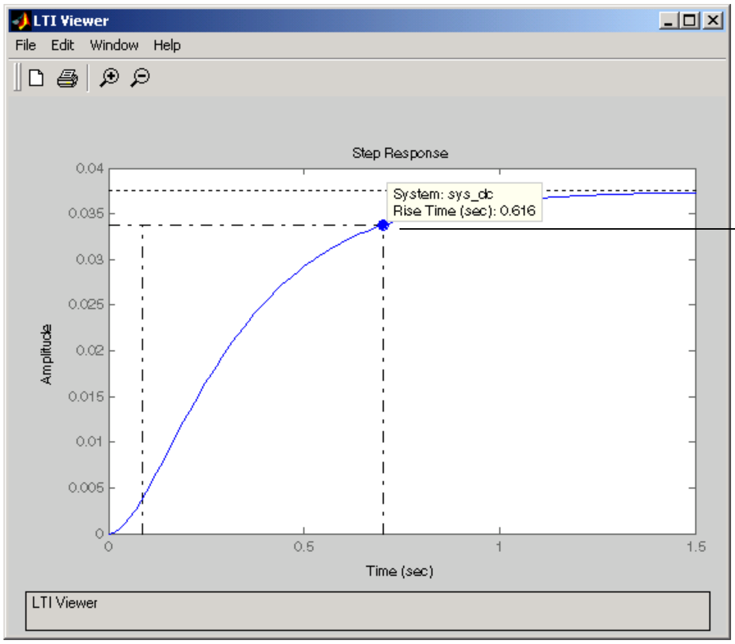
The LTI Viewer calculates and displays the rise time for the step response.


LTI Viewer
File Edit Window Help
Step Response
Amplitude
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
0.5
1
1.5
Time (sec)
LTI Viewer
The dot marks the 90% value for the rise time.
This line shows when the response reaches 10% of its steady-state value.
This line shows when the response reaches 90% of its steady-state value.

DC Motor Step Response with the Rise Time Displayed

To display the values of any plot characteristic marked on a plot, place your mouse on the blue dot that marks the characteristic. This opens a *data marker* with the relevant information displayed. To make the marker persistent, left-click the blue dot.

For example, this figure shows the rise time value for the DC motor step response.



Place your cursor over the blue dot to display a data marker with the system name and the 90% rise time value.

Left-click the blue dot to make the data marker persistent.

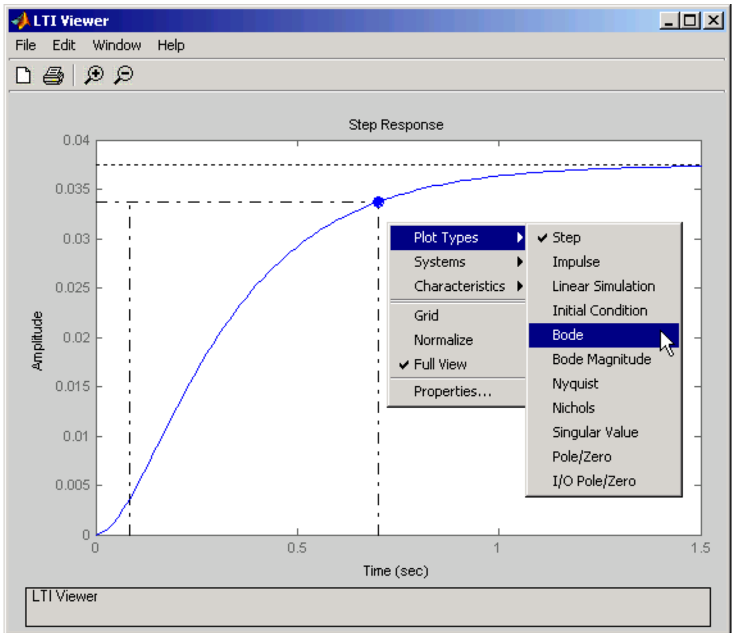
Using Your Mouse to Get the Rise Time Values

Note that you can left-click anywhere on a particular plot line to see the response values of that plot at that point. You must either place your cursor over the blue dot or left-click, however, if you want to see the rise time value.

For more information about data markers, see “Data Markers” on page 3-22.

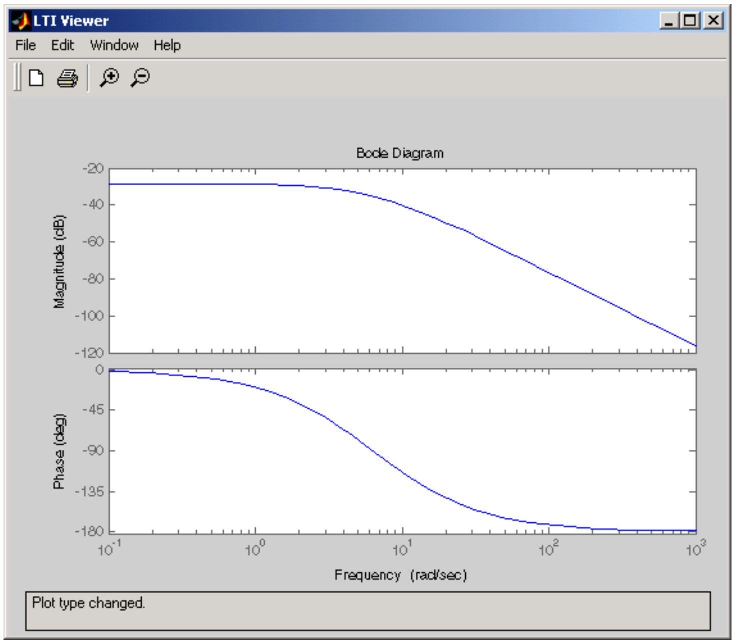
Changing Plot Type

You can view other plots using the right-click menus in the LTI Viewer. For example, if you want to see the open loop Bode plots for the DC motor model, select **Plot Type** and then **Bode** from the right-click menu.



Changing the Step Response to a Bode Plot

Selecting **Bode** changes the step response to a Bode plot for the DC motor model.

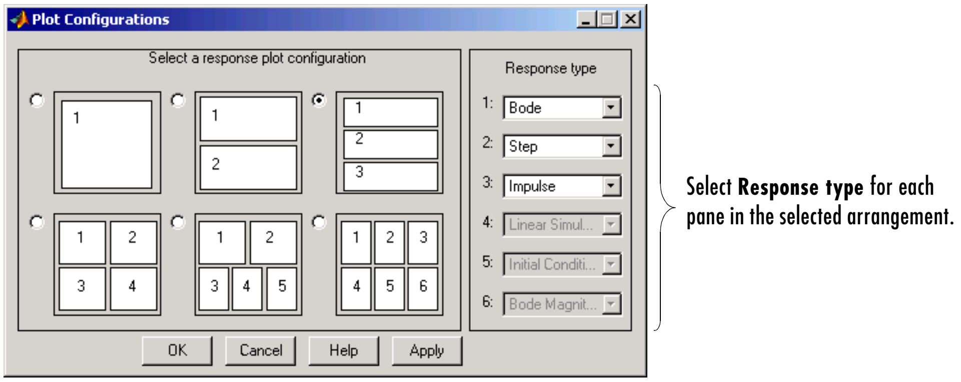


Plot type changed.

Bode Plot for the DC Motor Model

Showing Multiple Response Types

If you want to see, for example, both a step response and a Bode plot at the same time, you have to reconfigure the LTI Viewer. To view different response types in a single LTI Viewer, select **Plot Configuration**s under the **Edit** menu. This opens the **Plot Configurations** dialog box.



Using the Plot Configurations Dialog Box to Reconfigure the LTI Viewer

You can select up to six plots in one viewer. Choose the **Response type** for each plot area from the right side menus. There are nine available plot types:

- Step
- Impulse
- Bode (magnitude and phase)
- Bode Magnitude (only)
- Nyquist
- Nichols
- Sigma
- Pole/Zero
- I/O pole/zero

Comparing Multiple Models

This section shows you how to import and manipulate multiple models in one LTI Viewer. For example, if you have designed a set of compensators to control a system, you can compare the closed-loop step responses and Bode plots using the LTI Viewer.

A sample set of closed-loop transfer function models is included (along with some other models) in the MAT-file `ltiexamples.mat`. Type

```
load ltiexamples
```

to load the provided transfer functions. The three closed-loop transfer function models, `Gcl1`, `Gcl2`, and `Gcl3`, are for a satellite attitude controller.

In this example, you analyze the response plots of the `Gcl1` and `Gcl2` transfer functions.

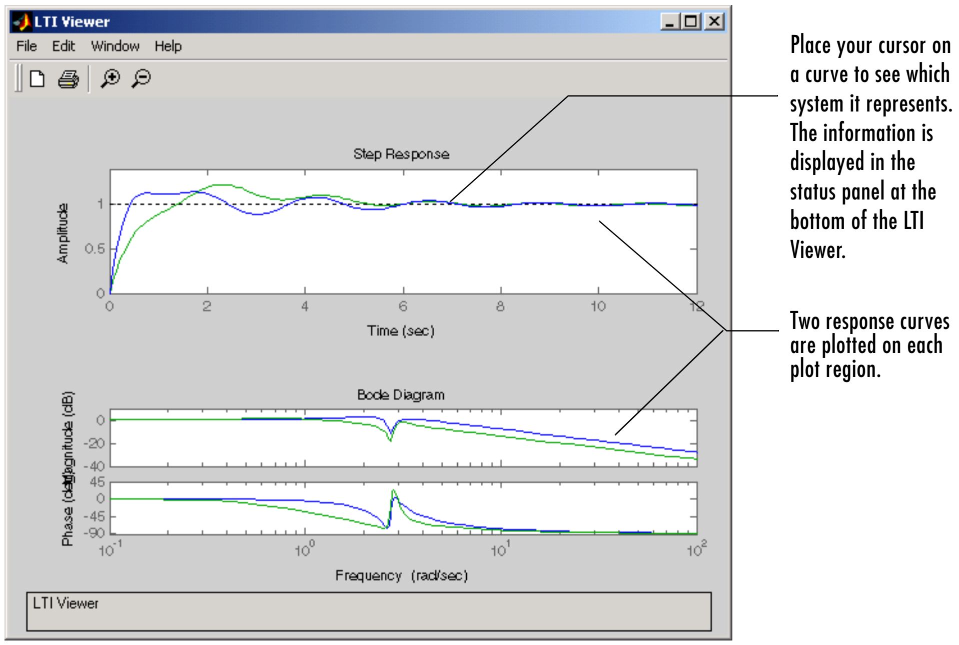
Initializing the LTI Viewer with Multiple Plots

To load the two models `Gcl1` and `Gcl2` into the LTI Viewer, select **Import** under the **File** menu and select the desired models in the LTI Browser. See "Importing Models into the LTI Viewer" on page 3-4 for a description of how to select groups of models. If necessary, you can reconfigure the viewer to display both the step responses and the Bode plots of the two systems using the **Viewer Configuration** dialog box. See "Showing Multiple Response Types" on page 3-12 for a discussion of this feature.

Alternatively, you can open an LTI Viewer with both systems and both the step responses and Bode plots displayed. To do this, type

```
ltiview({'step';'bode'},Gcl1,Gcl2)
```

Either approach opens the following LTI Viewer.


LTI Viewer
File Edit Window Help
Step Response
Amplitude
Time (sec)
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
LTI Viewer
Place your cursor on a curve to see which system it represents. The information is displayed in the status panel at the bottom of the LTI Viewer.
Two response curves are plotted on each plot region.

Multiple Response Plots in a Single LTI Viewer

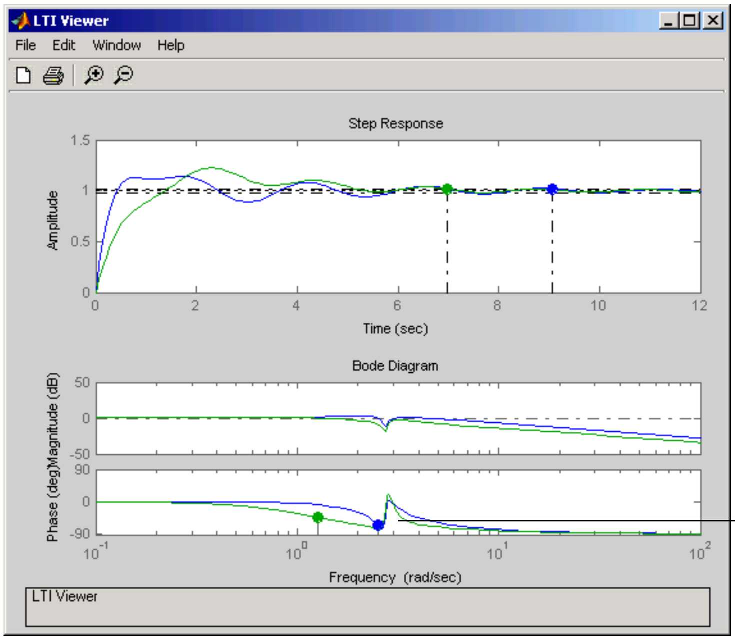
Inspecting Response Characteristics

To mark the settling time on the step responses presented in this example, do the following:

- Right-click anywhere in the plot region of the step response plots. This opens the right-click menu list in the plot region.
- Place your mouse pointer on the **Characteristics** menu item, and select **Settling Time** with your left mouse button.

To mark the stability margins of the Bode plot in this example, open the right-click menu and select **Stability Margins (Minimum Crossing)** under the **Characteristics** menu item.

Your LTI Viewer should now look like this.



Multiple Plots with Response Characteristics Added

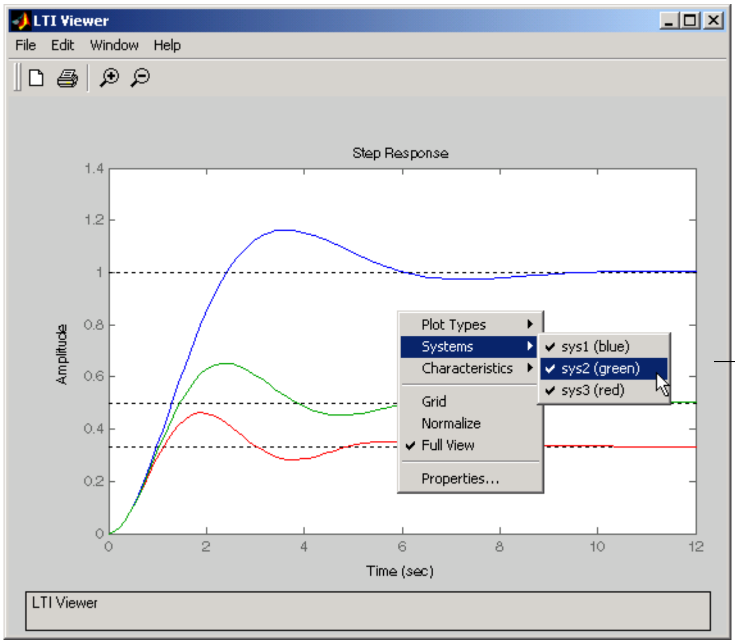
The minimum stability margins, meaning the smallest magnitude phase and gain margins, are displayed as green and blue markers on the Bode phase diagram. If you want to see all the gain and phase margins of a system, select **Stability Margins (All Crossings)** under the **Characteristics** menu item.

Toggling Model Visibility

If you have imported more than one model, you can select and clear the models to plot in the LTI Viewer using right-click menus. For example, if you import the following three models into the viewer, you can choose to view any combination of the three you want.

```
s=tf('s');
sys1=1/(s^2+s+1);
sys2=1/(s^2+s+2);
sys3=1/(s^2+s+3);
```

This figure shows how to clear the second of the three models using right-click menu options.



Select/clear the systems you want to add or remove from the LTI Viewer under **Systems** in the right-click menu. The menu lists only systems that you have imported into the LTI Viewer.

Using Right-Click Menus to Select/Clear Plotted Systems

The **Systems** menu lists all the imported models. A system is selected if a check mark is visible to the left of the system name.

Functions for Time and Frequency Response

The Control System Toolbox provides the LTI Viewer, a GUI that is suitable for a wide range of applications. There are situations, however, where you may want a more open and extensible environment. The Control System Toolbox provides a set of functions that provide the basic time and frequency domain analysis plots used in control system engineering. These functions apply to any kind of linear model (continuous or discrete, SISO or MIMO, or arrays of models). You can only apply the frequency domain analysis functions to FRD models.

Use the LTI Viewer when a GUI-driven environment is desirable. On the other hand, use functions when you want customized plots. If you want to include data unrelated to your models, you must use functions instead of the LTI Viewer (which only plots model data).

The next sections discuss time and frequency response functions and how to use these functions to create customized plots of linear model responses.

Time and Frequency Responses

Time responses investigate the time-domain transient behavior of linear models for particular classes of inputs and disturbances. You can determine such system characteristics as rise time, settling time, overshoot, and steady-state error from the time response. The Control System Toolbox provides functions for step response, impulse response, initial condition response, and general linear simulations. For example, you can simulate the response to white noise inputs using `lsim` and the MATLAB function `randn`.

In addition to time-domain analysis, the Control System Toolbox provides functions for frequency-domain analysis using the following standard plots:

- Bode
- Nichols
- Nyquist
- Singular value

This table lists available time and frequency response functions and their use.

Functions for Frequency and Time Response

Functions	**Description**
`bode`	Bode plot
`evalfr`	Computes the frequency response at a single complex frequency (not for FRD models)
`freqresp`	Computes the frequency response for a set of frequencies
`gensig`	Input signal generator (for `lsim`)
`impulse`	Impulse response plot
`initial`	Initial condition response plot
`iopzmap`	Pole-zero map for each I/O pair of an LTI model
`lsim`	Simulation of response to arbitrary inputs
`margin`	Computes and plots gain and phase margins
`nichols`	Nichols plot
`nyquist`	Nyquist plot
`pzmap`	Pole-zero map
`step`	Step response plot

These functions can be applied to single linear models or LTI arrays.

The functions `step`, `impulse`, and `initial` automatically generate an appropriate simulation horizon for the time response plots. Their syntax is

```
step(model_name)
impulse(model_name)
initial(model_name,x0)    % x0 = initial state vector
```

where `model_name` is any continuous or discrete LTI model or LTI array.

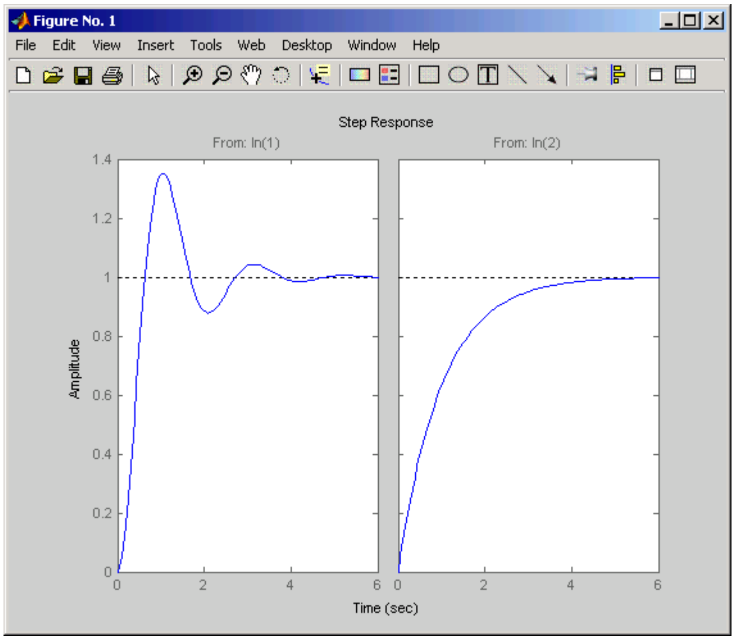
Frequency-domain plots automatically generate an appropriate frequency range as well.

Plotting MIMO Model Responses

For MIMO models, time and frequency response functions produce an array of plots with one plot per I/O channel (or per output for `initial` and `lsim`). For example,

```
h = [tf(10,[1 2 10]) , tf(1,[1 1])]
step(h)
```

produces the following plot.



Step Responses for a MIMO Model

The simulation horizon is automatically determined based on the model dynamics. You can override this automatic mode by specifying a final time,

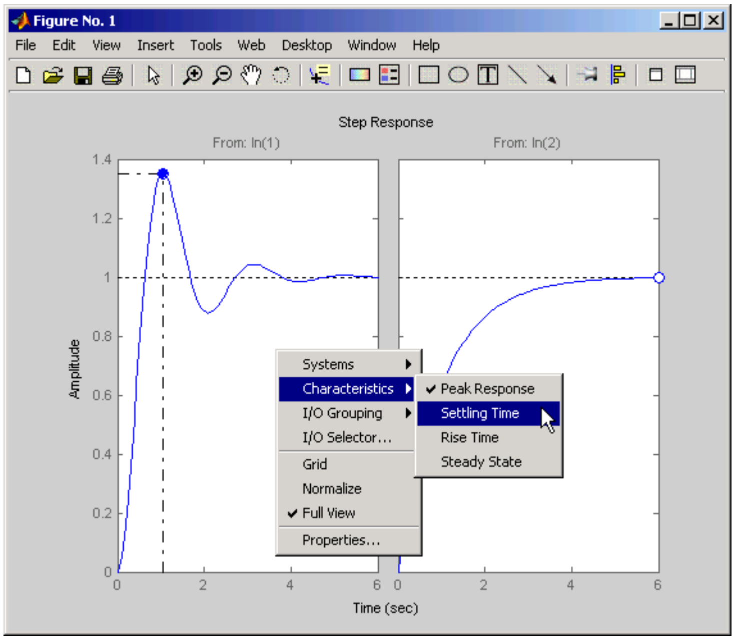
```
step(h,10) % Simulates from 0 to 10 seconds
```

or a vector of evenly spaced time samples.

```
t = 0:0.01:10   % Time samples spaced every 0.01 second
step(h,t)
```

Right-Click Menus

All the time and frequency response functions provide right-click menus that allow you to customize your plots. This figure shows the plots from the figure titled “Step Responses for a MIMO Model” on page 3-19, with the right-click menu open.



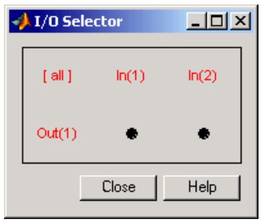
Using the Right-Click Menu in a Step Response Plot

The options you can select include

- **Systems** — Select or clear any models that you included when you created the response plot.
- **Characteristics** — Add information about the plot. The characteristics available change from plot to plot. For example, Bode plots have stability

margins available, but step responses have rise time and steady-state values available.

- **Axes Grouping** — Change the grouping of your plots. Available options are All, None, Inputs, and Outputs. You can group all the plots together, place each in a separate plot region (none), or group the inputs or outputs together.
- **I/O Selector** — Open the **I/O Selector** window.



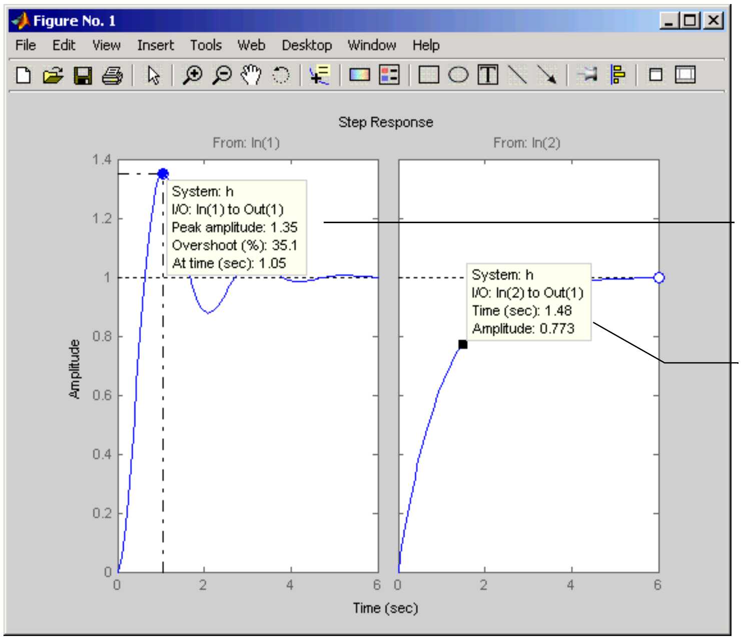
Use this window to select/clear the inputs and outputs to plot.

- **Normalize** — Scale responses to fit the view (only available for time-domain plot types).
- **Full View** — Use automatic limits to make the entire curve visible.
- **Grid** — Add grids to your plots.
- **Properties** — Open the Property Editor, which you can use to customize various attributes of your plot. See "Customization" in the Control System Toolbox online documentation for a full description of the Property Editor.

Alternatively, you can open the Property Editor by double-clicking in an empty region of the response plot.

Data Markers

In addition to right-click menus, the Control System Toolbox provides plot data markers. These allow you to identify key data points on your plots. This figure, using the same plot as in “Step Responses for a MIMO Model” on page 3-19, shows markers on the plots.



Place your cursor on an active characteristic, shown as a blue dot here, to see its name and numerical value.

Left-click anywhere on a plot to see the system name, I/O labels, time, and value at that point.

Using Plot Markers to Identify Data Points

You can move a data marker by

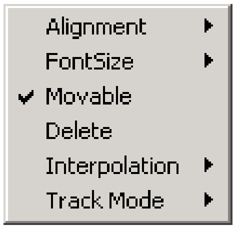
- Grabbing the black square located at the corner of the marker
- Dragging the marker with your mouse

The time and amplitude values will change as you move the marker. This does not apply to markers that display plot characteristics (e.g., peak value or rise time). In the case of plot characteristic data markers, you can view them by placing your cursor over the dot that represents the active characteristic. To make the data marker persistent, left-click the marker.

Note Data markers do not apply to the SISO Design Tool, which displays data about plot characteristics in the status panel at the bottom of the SISO Design Tool window.

Right-Click Menus

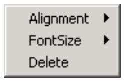
Right-click any data marker to open a property menu for the marker.



Property options for the marker include

- **Alignment** — Change the position of the marker. Available options are top-right, top-left, bottom-right, and bottom-left.
- **Font Size** — Change the font size.
- **Movable** — By default, you can move data markers by clicking and dragging. Clearing **Movable** forces the marker to remain at a fixed data point.
- **Delete** — Delete the selected marker. Alternatively, left-click anywhere in the empty plot region to delete all markers in the plot.
- **Interpolation** — By default, data markers linearly interpolate between points along the plotted curve. Select **None** to force the markers to snap to nearest points along the plotted curve.
- **Track Mode** — The default is to track *x*- and *y*-values. You can choose to track only *x*- or *y*-values as well.

Since characteristic data markers are by definition fixed, the right-click menus for them have fewer options.



These options work the same as they do for the full right-click menu.

Plotting and Comparing Multiple Systems

You can use the command-line response-plotting functions to plot the response of continuous and discrete linear models on a single plot. To do so, invoke the corresponding command-line function using the list `sys1`,..., `sysN` of models as the inputs.

```
step(sys1,sys2,...,sysN)
impulse(sys1,sys2,...,sysN)
...
bode(sys1,sys2,...,sysN)
nichols(sys1,sys2,...,sysN)
...
```

All models in the argument lists of any of the response plotting functions (except for `sigma`) must have the same number of inputs and outputs. To differentiate the plots easily, you can also specify a distinctive color/linestyle/marker for each system just as you would with the `plot` command. For example,

```
bode(sys1,'r',sys2,'y--',sys3,'gx')
```

plots `sys1` with solid red lines, `sys2` with yellow dashed lines, and `sys3` with green `x` markers.

You can plot responses of multiple models on the same plot. These models do not need to be all continuous-time or all discrete-time.

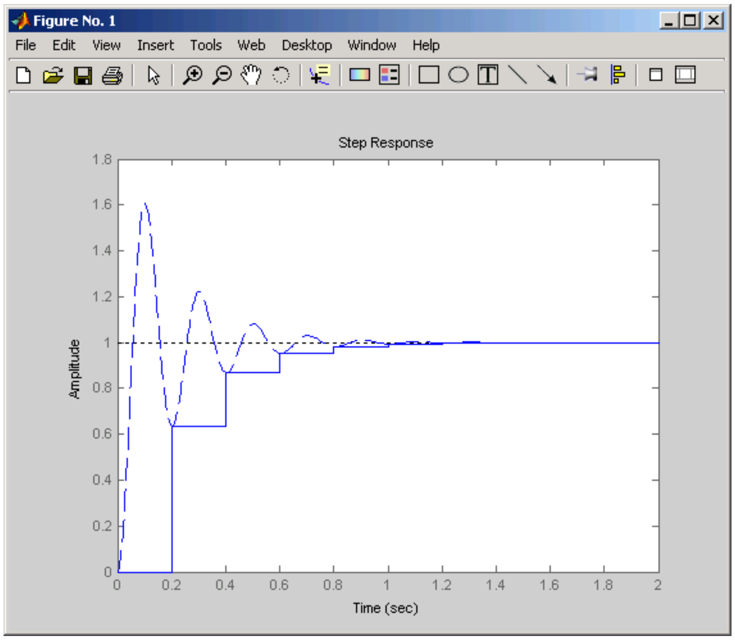
Example: Comparing Continuous and Discretized Systems

The following example compares a continuous model with its zero-order-hold discretization.

```
sysc = tf(1000,[1 10 1000])
sysd = c2d(sysc,0.2)       % ZOH sampled at 0.2 second

step(sysc,'--',sysd,'-')   % Compare step responses
```

These commands produce the plot shown below.

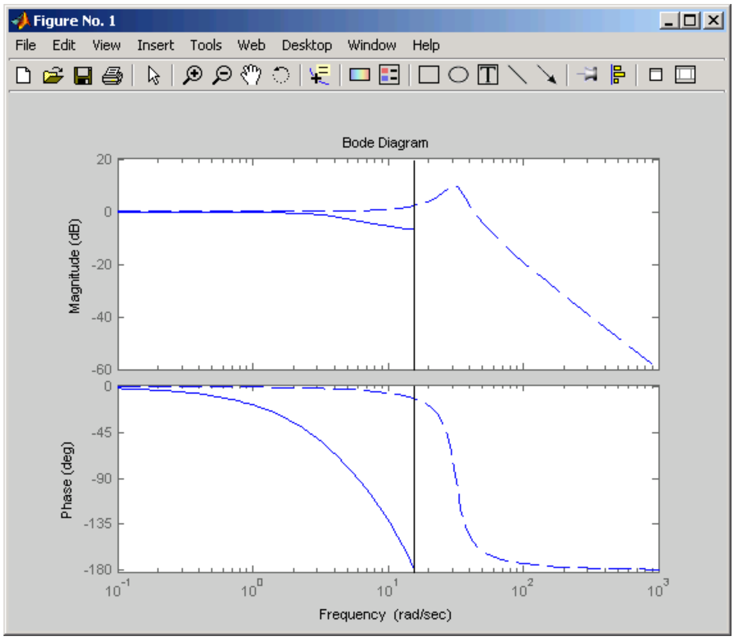


Comparison of a Continuous Model to Its Discretized Version

Use this command to compare the Bode plots of the two systems.

```
bode(sysc,'--',sysd,'-')     % Compare Bode responses
```

The Control System Toolbox creates this plot.



Comparison of Bode Plots for a Continuous Model and Its Discretized Version

A comparison of the continuous and discretized responses reveals a drastic undersampling of the continuous-time system. Specifically, there are hidden oscillations in the discretized time response and aliasing conceals the continuous-time resonance near 30 rad/s.

Creating Custom Plots

Time and frequency response commands are useful for creating custom plots. You can mix model response plots with other data views using response commands together with `plot`, `subplot`, and `hold`.

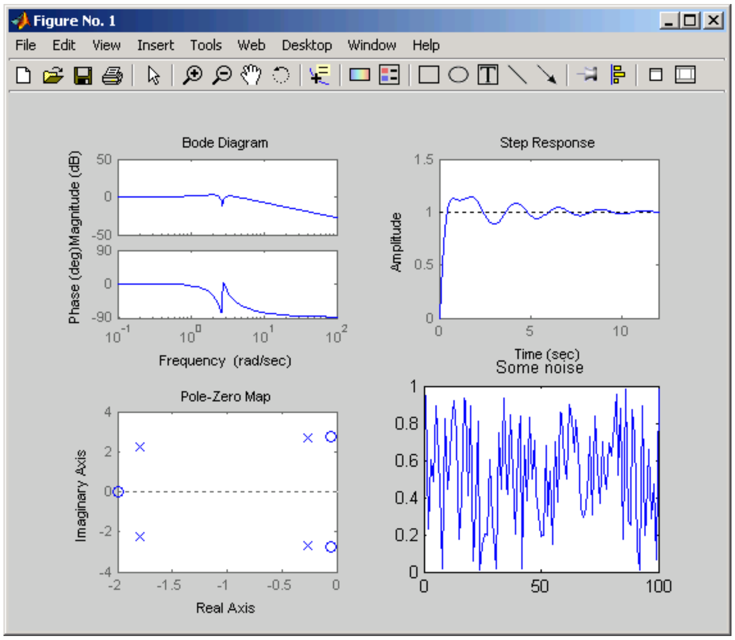
Example: Custom Plots

For example, the following sequence of commands displays the Bode plot, step response, pole/zero map, and some additional data in a single figure window.

```
h = tf([4 8.4 30.8 60],[1 4.12 17.4 30.8 60]);
```

```
subplot(221)
bode(h)
subplot(222)
step(h)
subplot(223)
pzmap(h)
subplot(224)
plot(rand(1, 100))     % Any data can go here
title('Some noise')
```

Your plot should look similar to this illustration.



Example of Model and Nonmodel Data Plotted in One Window

For information about `plot`, `subplot`, `hold`, and other options for plotting general data, see "Basic Plots and Graphs" in the MATLAB Function Reference. These documents are available in the MATLAB online help.

Note Each of the plots generated by response analysis functions in the preceding figure (`bode`, `step`, and `pzmap`) has its own right-click menu (similar to those in the LTI Viewer). To activate the right-click menus, place your mouse in the plot region and right-click. The menu contents depend on what type of plot you have selected.

Designing Compensators

The SISO Design Tool

The SISO Design Tool is a graphical user interface (GUI) that facilitates the design of compensators for single-input, single-output feedback loops. The SISO Design Tool allows you to iterate rapidly on your designs and perform the following tasks:

- Manipulate closed-loop dynamics using root locus techniques.
- Shape open-loop Bode responses.
- Add compensator poles and zeros.
- Add and tune lead/lag networks and notch filters.
- Inspect closed-loop responses (using the LTI Viewer).
- Adjust phase and gain margins.
- Convert models between discrete and continuous time.

Opening the SISO Design Tool

This section shows how to open the SISO Design Tool with the DC motor example developed in Chapter 2, "Building Models."

If you have not built the DC motor model, type

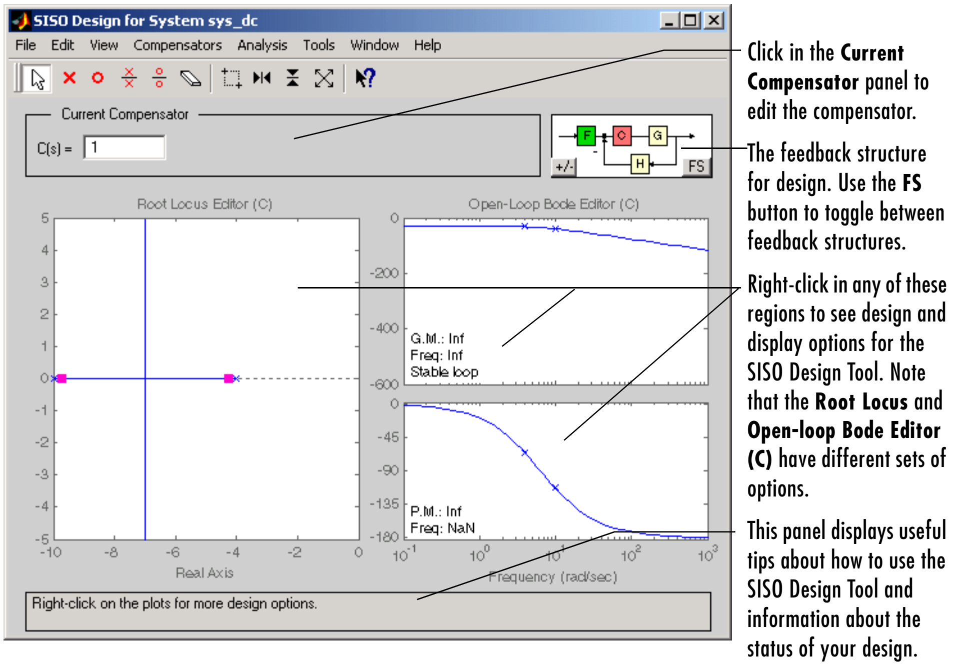
```
load ltiexamples
```

at the MATLAB prompt. This loads a collection of linear models, including the DC motor. To open the SISO Design Tool and import the DC motor, type

```
sisotool(sys_dc)
```

at the MATLAB prompt.

This command opens the SISO Design Tool with the root locus and open-loop Bode diagrams for the DC motor plotted by default.


SISO Design for System sys_dc
File Edit View Compensators Analysis Tools Window Help
Current Compensator
C(s) = 1
F
C
G
H
+/-
FS
Root Locus Editor (C)
Real Axis
Open-Loop Bode Editor (C)
G.M.: Inf
Freq: Inf
Stable loop
P.M.: Inf
Freq: NaN
Frequency (rad/sec)
Right-click on the plots for more design options.
Click in the **Current Compensator** panel to edit the compensator.
The feedback structure for design. Use the **FS** button to toggle between feedback structures.
Right-click in any of these regions to see design and display options for the SISO Design Tool. Note that the **Root Locus** and **Open-loop Bode Editor (C)** have different sets of options.
This panel displays useful tips about how to use the SISO Design Tool and information about the status of your design.

SISO Design Tool with the DC Motor Example

The SISO Design Tool displays

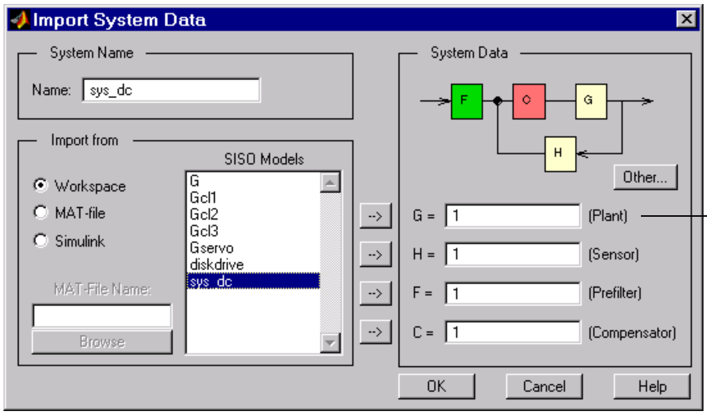
- Poles as x's
- Zeros as o's
- Gain and phase margins (by default) in the lower left corners of the Bode magnitude and phase plots

Importing Models into the SISO Design Tool

If you type

```
sisotool
```

at the MATLAB prompt, an empty SISO Design Tool opens. You can import the DC motor model by selecting **Import** under the **File** menu. This opens the **Import System Data** dialog box, which is shown below.



Import System Data

System Name

Name: sys_dc

Import from

[✓ Workspace] MAT-file Simulink

SISO Models: G, Gcl1, Gcl2, Gcl3, Gservo, diskdrive, [✓ sys_dc]

MAT-File Name:

Browse

System Data

G = 1 (Plant)

H = 1 (Sensor)

F = 1 (Prefilter)

C = 1 (Compensator)

Place the DC motor model (sys_dc) in the field marked **G** (Plant).

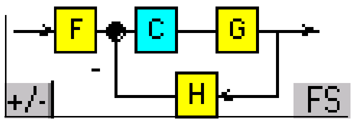
Importing the DC Motor Model into the SISO Design Tool

Follow these steps to import the DC motor model:

1. Select `sys_dc` under **SISO Models**.
2. Place it into the **G** field under **Design Model** by clicking the right arrow button to the left of **G**.
3. Click **OK**.

Feedback Structure

The SISO Design Tool by default assumes that the compensator is in the forward path, i.e., that the feedback structure looks like this figure.



The Default Feedback Structure — Compensator in the Forward Path

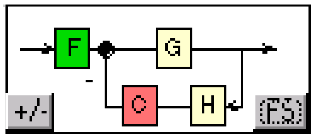
In this figure, the lettered boxes represent the following:

- **G** — plant
- **H** — sensor dynamics
- **F** — prefilter
- **C** — compensator

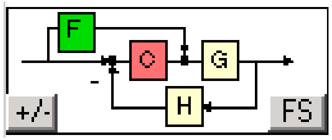
The default values for **F**, **H**, and **C** are all 1. Note that this means that by default, the compensator has unity gain. **G** contains the DC motor model, `sys_dc`.

Alternative Feedback Structures

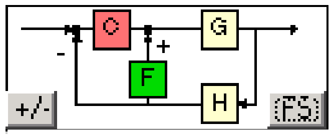
Clicking the **FS** button cycles through the default feedback structure and several other feedback structures. This figure shows the alternate feedback stuctures.



Compensator in the feedback path



Feedforward controller



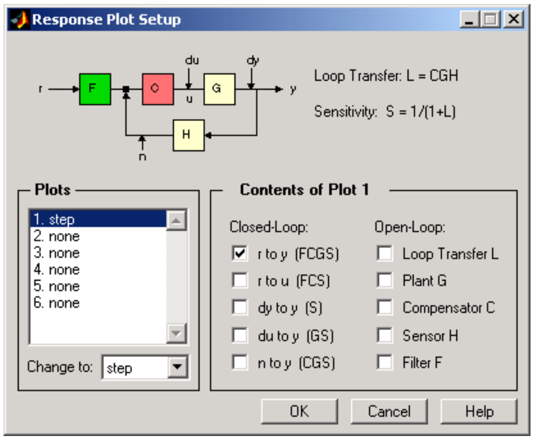
Cascade configuration with filter F in the minor loop

Alternate Feedback Structures

Loop Responses

As you iterate on a compensator design, you may find it convenient to be able to examine the various loop responses (for example, step or impulse responses).

To view, for example, the closed-loop step response, select **Other Loop Response** from the **Analysis** menu. This opens the **Response Plot Setup** window with the default setting of closed-loop step response from *r*, the reference signal, to *y*, the output signal.



Loop Transfer: L = CGH

Sensitivity: S = 1/(1+L)

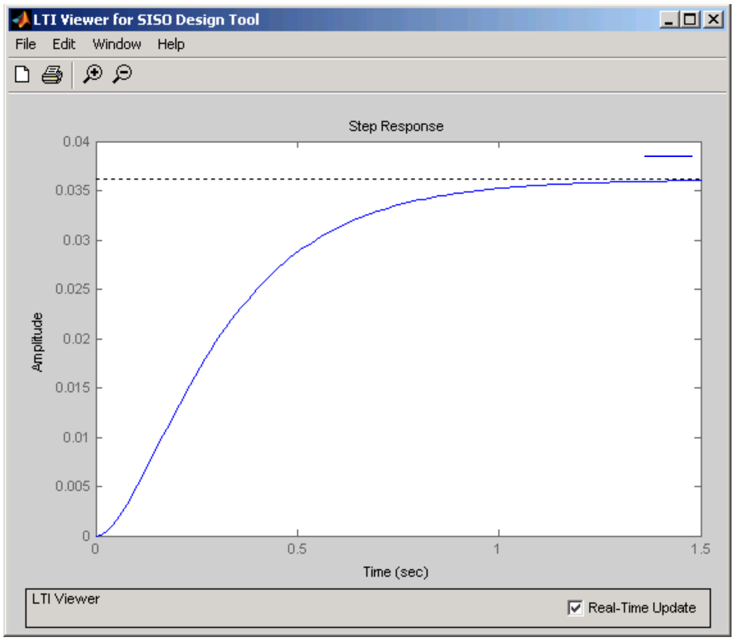
Plots: 1. step, 2. none, 3. none, 4. none, 5. none, 6. none

Change to: step

Closed-Loop:	Open-Loop:
✓ r to y (FCGS)	☐ Loop Transfer L
☐ r to u (FCS)	☐ Plant G
☐ dy to y (S)	☐ Compensator C
☐ du to y (GS)	☐ Sensor H
☐ n to y (CGS)	☐ Filter F

Click **OK** to open an LTI Viewer with the closed-loop step response of the DC motor. For instructions on how to operate the LTI Viewer, see “LTI Viewer” on page 3-2.

This figure shows the resulting plot.



LTI Viewer Showing the Step Response for the DC Motor

As this plot shows, the step response of the DC motor is about 1.5 seconds, which is too slow for many applications. Also, there is a large steady-state error. The following sections show how to use Bode diagram techniques for improving the response time and steady-state error of the DC motor step response.

As you iterate on a design, the LTI Viewer associated with your SISO Design Tool will automatically update the response plots you have chosen.

Bode Diagram Design

One technique for compensator design is to work with Bode diagrams of the open-loop response (loop shaping). Using Bode diagrams, you can design to gain and phase margin specifications, adjust the bandwidth, and add notch filters for disturbance rejection.

Example: DC Motor

The following sections use the DC motor example to show how create a compensator using Bode diagram design techniques. From "SISO Example: the DC Motor" on page 2-4, the transfer function of the DC motor is

```
Transfer function:
       1.5
------------------
s^2 + 14 s + 40.02
```

For this example, the design criteria are as follows:

- Rise time of less than 0.5 second
- Steady-state error of less than 5%
- Overshoot of less than 10%
- Gain margin greater than 20 dB
- Phase margin greater than 40 degrees

Adjusting the Compensator Gain

The preceding figure shows that the closed-loop step response is too slow. The simplest approach to speeding up the response is to increase the gain of the compensator. To increase the gain:

1. Move the mouse pointer over the Bode magnitude line. Notice how the pointer becomes a hand.
2. Grab the Bode magnitude line by holding down the left mouse button when the hand appears.
3. Drag the Bode plot line upward.

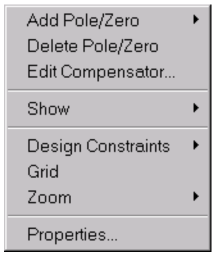
4 Release the mouse button. The gain and poles change as the closed-loop set point is recomputed.

The SISO Design Tool calculates the compensator gain, and the value appears in the **C(s)** text box on the GUI.

Alternatively, you can set the gain by entering the desired value in the **C(s)** field in the **Current Compensator** panel.

Right-Click Menus

The SISO Design Tool has right-click menus available in any of the plot regions. The menus are customized for each plot type; open the Bode magnitude menu by right-clicking your mouse in the white space of the Bode magnitude plot. This menu appears.



Right-Click Menu for the Bode Magnitude Plot

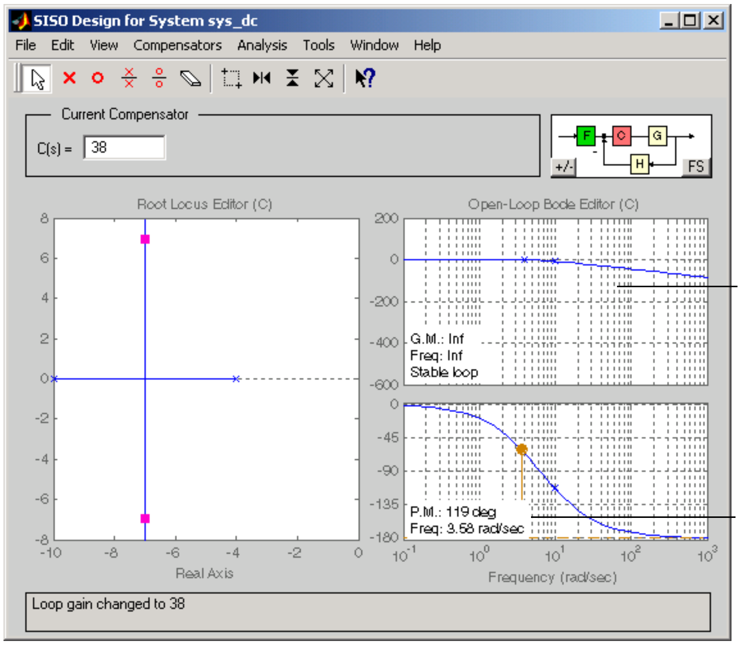
The right-click menus contain numerous features. The DC motor example makes use of many of the available features; for a complete discussion of the right-click menus, see the online help for the SISO Design Tool in GUI Reference.

Adjusting the Bandwidth

Since the design requirements include a 0.5 second rise time, try setting the gain so that the DC crossover frequency is about 3 rad/s. The rationale for setting the bandwidth to 3 rad/s is that, to a first-order approximation, this should correspond to about a 0.33 second time constant.

To make the crossover easier to see, select **Grid** from the right-click menu. This creates a grid for the Bode magnitude plot. Left-click on the Bode magnitude plot and drag the curve until you see the curve crossing over the 0 dB line (on the *y* axis) at 3 rad/s. This changes both the SISO Design Tool display and the LTI Viewer step response.

This figure shows the SISO Design Tool.



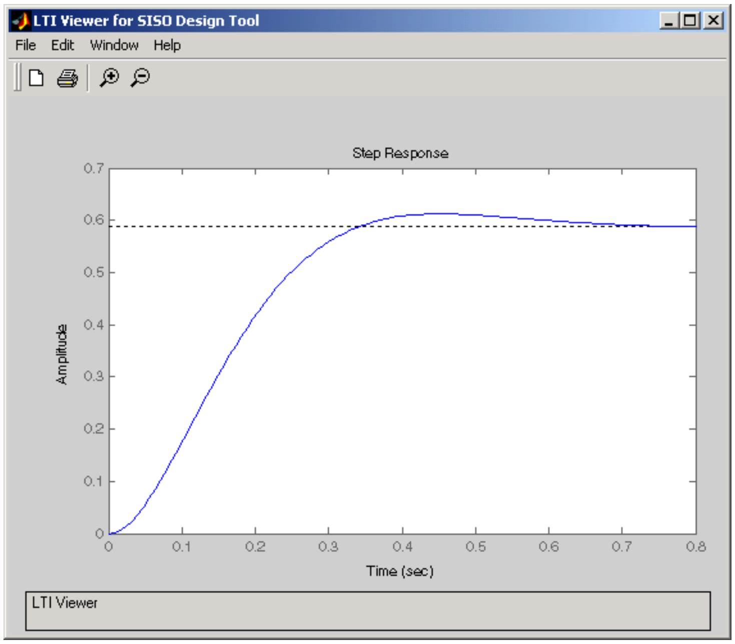
Use the hand to move the Bode magnitude plot up or down. The SISO Design Tool recalculates the compensator gain as you move the hand.

Increasing the Compensator gain changed the phase margin from infinity to 119°. The SISO Design Tool adds a brown stem to show the new phase margin.

Root Locus and Bode Plots for the DC Motor

For a crossover at 3 rad/s, the compensator gain should be about 38. By default, the SISO Design Tool displays gain and phase margin information in the lower left-hand corners of the Bode diagrams. In the Bode magnitude plot, it also tells you if your closed-loop system is stable or unstable.

This plot shows the associated closed-loop step response in the LTI Viewer.

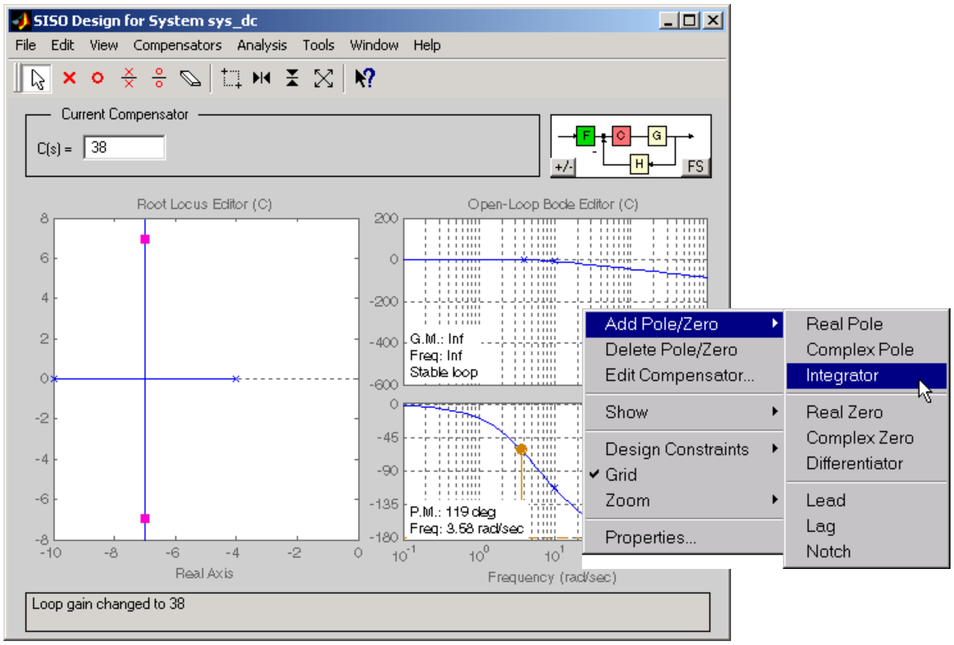


Closed-Loop Step Response for the DC Motor with a Compensator Gain = 38

The step response shows that the steady-state error and rise time have improved somewhat, but you must design a more sophisticated controller to meet all the design specifications, in particular, the steady-state error requirement.

Adding an Integrator

One way to eliminate steady-state error is to add an integrator. To do this, select **Add Pole/Zero** and then **Integrator** from the right-click menu. This figure shows the process.

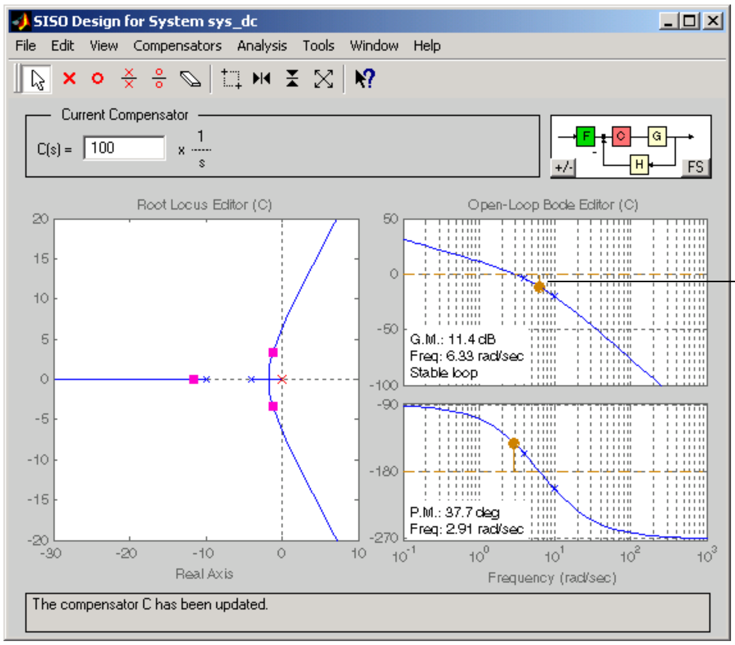


Loop gain changed to 38

Using the Right-Click Menu to Add an Integrator

Notice adding the integrator changed the crossover frequency of the system. Readjust the compensator gain to bring the crossover back to 3 rad/s; the gain should be about 100.

Once you have added the integrator and readjusted the compensator gain, the SISO Design Tool shows a red 'x' at the origin of the root locus plot.

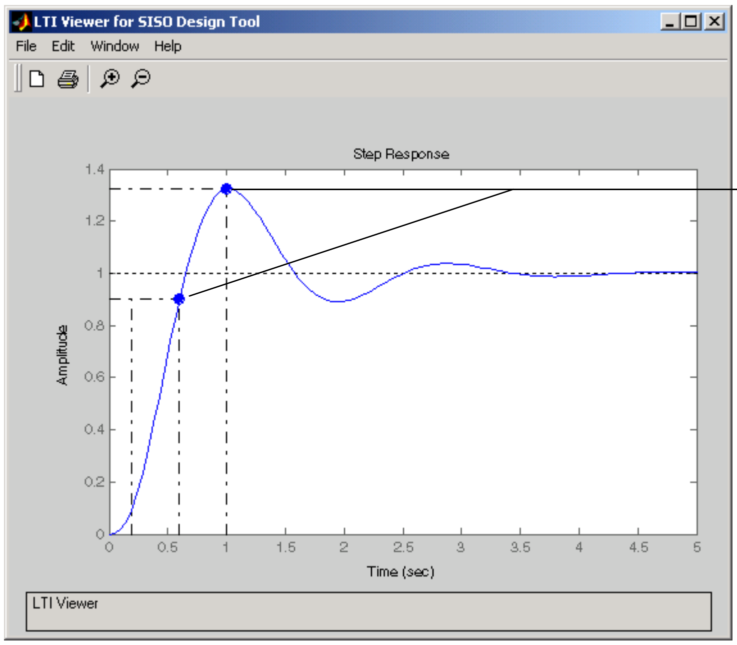


The compensator C has been updated.

Adding an integrator changed the gain margin from infinity to 11.5 dB. The SISO Design Tool displays the gain margin as a brown stem.

The SISO Design Tool Displays the Integrator on the Root Locus Plot

This figure shows the closed-loop step response.



Use the right-click menu to display the **Peak Response** and **Rise Time** (listed under **Characteristics**).

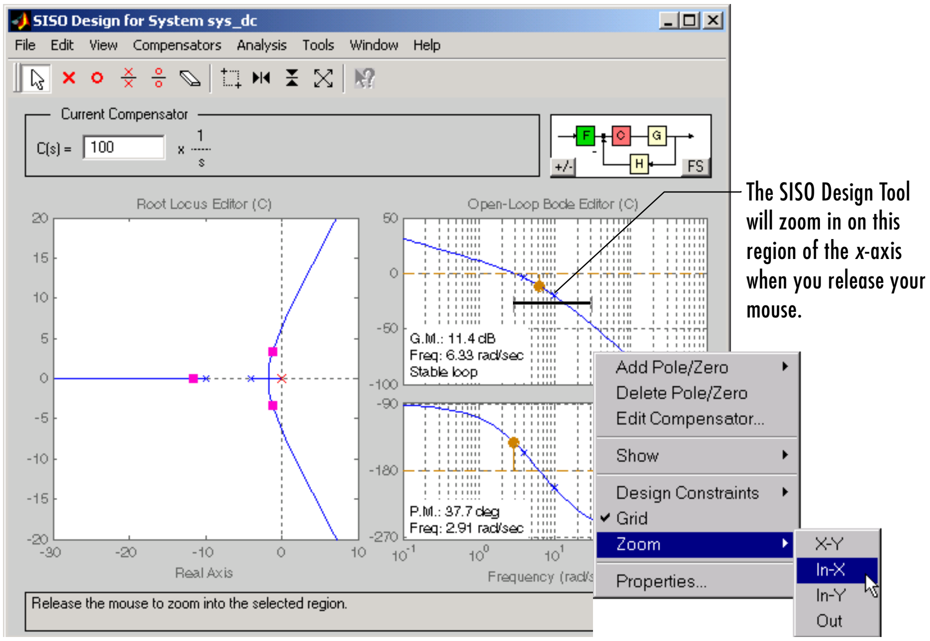
The Step Response for the DC Motor with an Integrator in the Compensator

The step response is settling around 1, which satisfies the steady-state error requirement. This is because the integrator forces the system to zero steady-state error. The figure shows, however, that the peak response is 1.3, or about 30% overshoot, and that the rise time is roughly 0.4 second. So a compensator consisting of an integrator and a gain is not enough to satisfy the design requirements, which require that the overshoot be less than 10%.

Adding a Lead Network

Part of the design requirements is a gain margin of 20 dB or greater and a phase margin of 40° or more. In the current compensator design, the gain margin is 11.5 dB and the phase margin is 38.1°, both of which fail to meet the design requirements. So two goals left are to shorten the rise time while improving the stability margins. One approach is to increase the gain to speed up the response, but the system is already underdamped, and increasing the gain will decrease the stability margin as well. You might try experimenting with the compensator gain to verify this. The only option left is to add dynamics to the compensator.

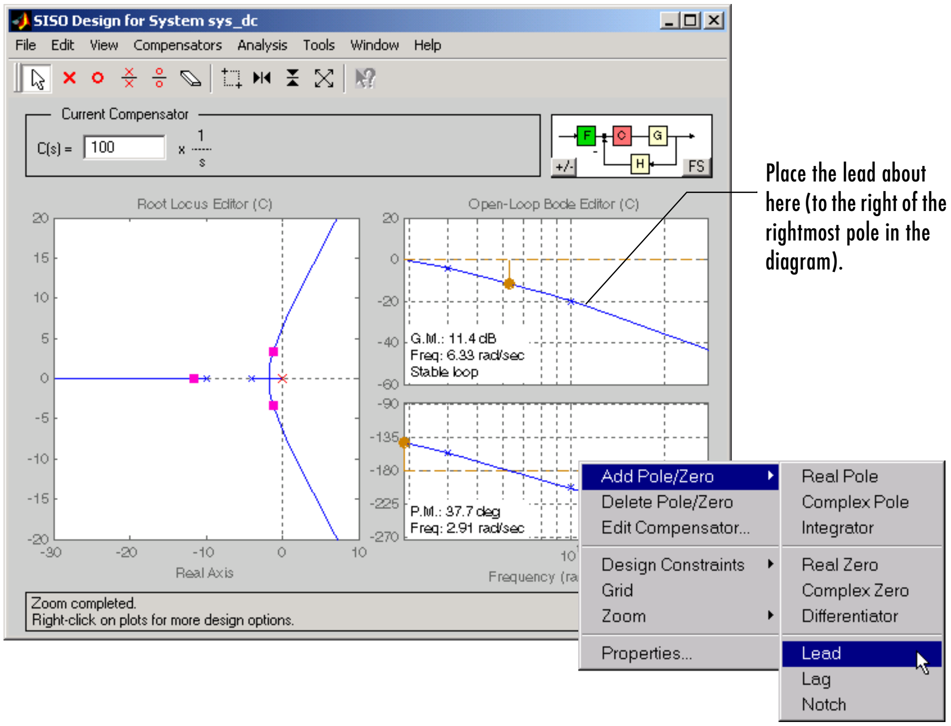
One possible solution is to add a lead network to the compensator. To make this easier to do on the diagram, zoom in on the x-axis. First, select **Zoom In-X** from the right-click menu; then select a region of the Bode magnitude plot by left-clicking and dragging your mouse. The range from 1 to about 50 rad/s is good. This figure shows the process.



Release the mouse to zoom into the selected region.

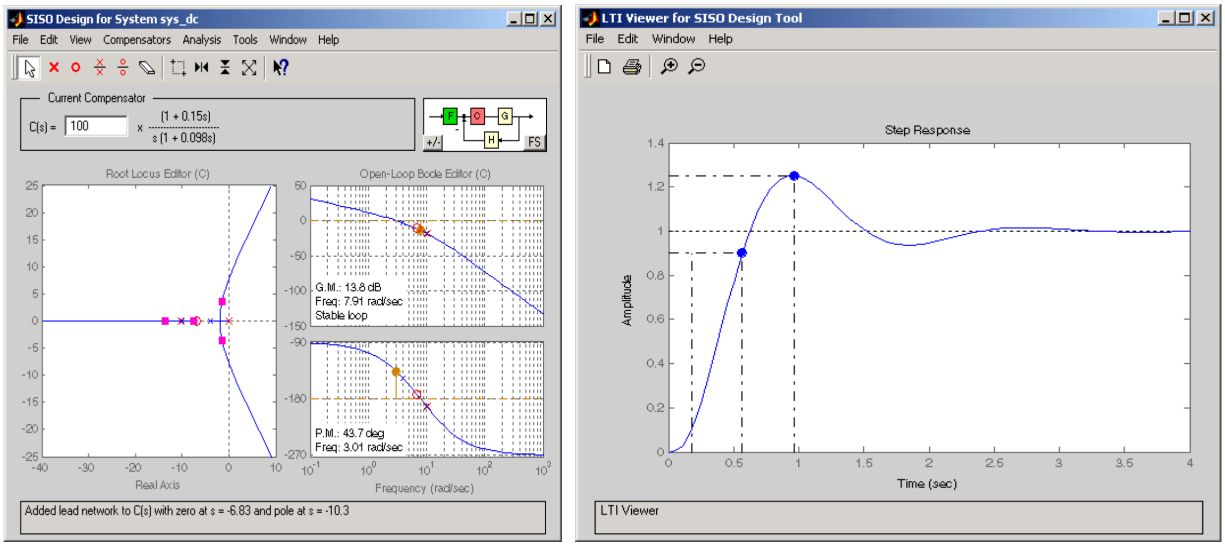
Zooming in on the X-Axis of the Bode Plots

To add the lead network, choose **Add Pole/Zero** and then **Lead** in the right-click menu for the Open-Loop Bode diagram. This figure shows the process of adding a lead network to your controller.



Adding a Lead Network to the DC Motor Compensator Using Right-Click Menus

Selecting a lead network causes your cursor to change to an ‘x.’ Position this ‘x’ on the Bode magnitude curve slightly to the right of the rightmost pole and click. Your SISO Design Tool and LTI Viewer plots should now look similar to these.



C(s) = 100 x (1 + 0.15s) / s (1 + 0.098s)

Added lead network to C(s) with zero at s = -6.83 and pole at s = -10.3

LTI Viewer

Root Locus, Bode, and Step Response Plots for the DC Motor with a Lead Network

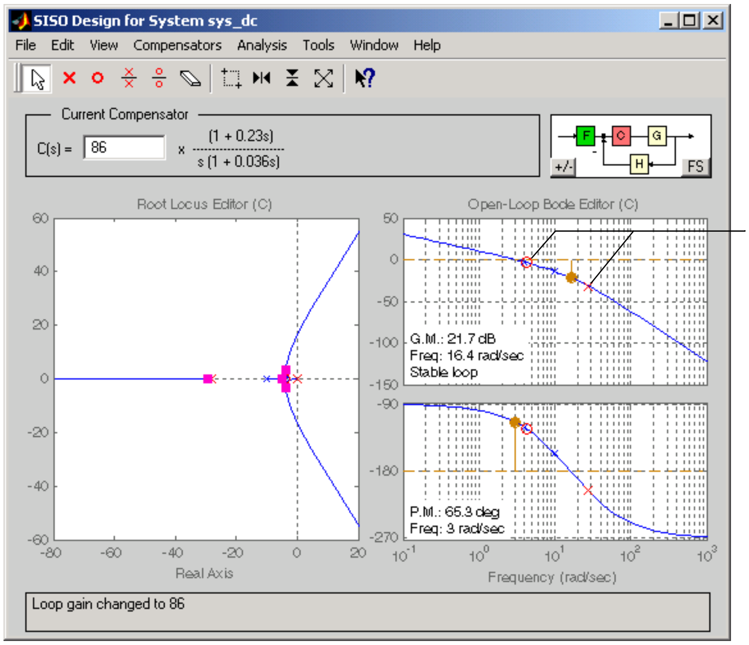
The Step Response plot shows that the rise time is now about 0.4 second and peak response is 1.25 rad/s (i.e., the overshoot is about 25%). Although the rise time meets the requirement, the overshoot is still too large, and the stability margins are still unacceptable, so you must tune the lead parameters.

Moving Compensator Poles and Zeros

To improve the response speed, move the lead network zero closer to the leftmost (slowest) pole of the DC motor plant (denoted by a blue ‘x’). To do this, just grab the zero and drag it with your mouse. Try positioning the zero near the slowest plant pole.

Now try moving the lead network pole to the right. Notice how the gain margin increases as you do this. You can also adjust the gain to increase the gain margin; grab the Bode magnitude curve and drag it downward with your mouse to reduce the gain and increase the gain margin.

As you tune these parameters, take a look at the LTI Viewer. You will see the closed-loop step response alter with each parameter change you make. The figure below shows the final values for a design that meets the specifications.


SISO Design for System sys_dc
File Edit View Compensators Analysis Tools Window Help
Current Compensator
F
C
G
H
+/-
FS
Root Locus Editor (C)
Real Axis
Open-Loop Bode Editor (C)
Frequency (rad/sec)
You can move red (compensator) poles and zeros by dragging them with your mouse.

C(s) = 86 x (1 + 0.23s) / s (1 + 0.036s)

G.M.: 21.7 dB
Freq: 16.4 rad/sec
Stable loop

P.M.: 65.3 deg
Freq: 3 rad/sec

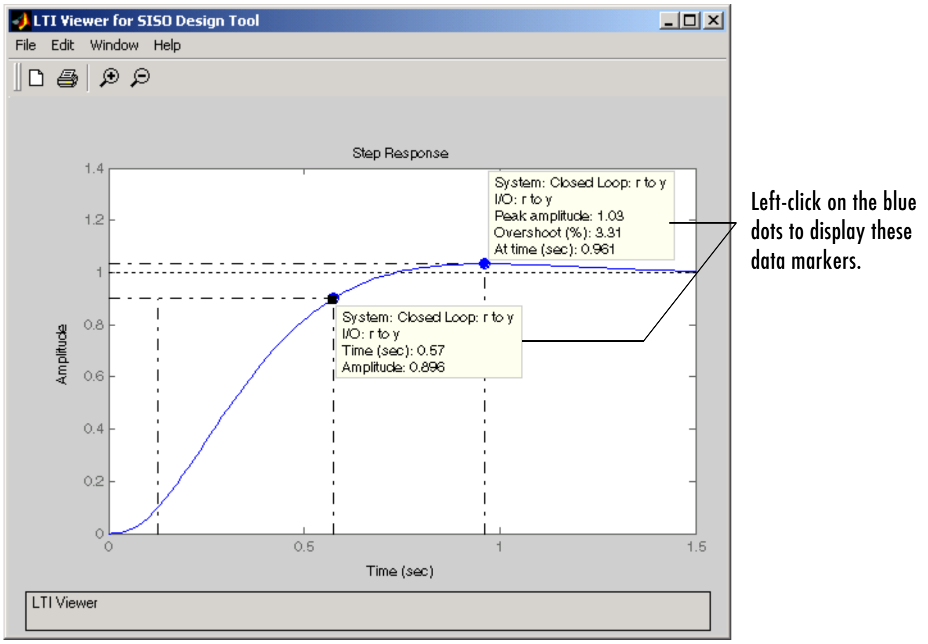
Loop gain changed to 86

Final Design Parameters for the DC Motor Compensator

The values for this final design are as follows:

- Poles at 0 and -28
- Zero at -4.3
- Gain = 84

You can use the **Edit Compensator** dialog box to specify the exact values. Double-click in the Current Compensator panel to open the window. This figure shows that the gain margin is 22 dB, and the phase margin is 66°. To see if the design meets the rise time and overshoot requirements, go to the closed-loop step response, right-click in an empty region of the plot, and select **Characteristics** and then **Rise Time** and **Peak Response**. This figure shows the plot with the rise time and overshoot denoted by large dots on the curve.



System: Closed Loop: r to y
I/O: r to y
Peak amplitude: 1.03
Overshoot (%): 3.31
At time (sec): 0.961

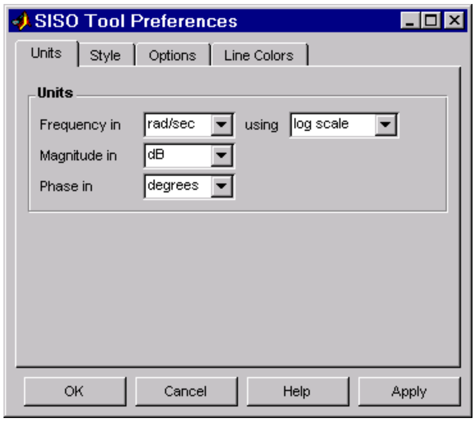
System: Closed Loop: r to y
I/O: r to y
Time (sec): 0.57
Amplitude: 0.896

Step Response for the Final Compensator Design

The step response shows that the rise time is 0.45 second, and the peak amplitude is 1.03 rad/s, or an overshoot of 3%. These results meet the design specifications.

Changing Units on a Plot

The Control System Toolbox provides editors for setting plot options. If you want, for example, to change the frequency units on all the Bode plots created in the SISO Design Tool from rad/s to Hertz, select **SISO Tool Preferences** under **Edit** in the menu bar. This opens the **SISO Tool Preferences** editor.



Units

Frequency in rad/sec using log scale

Magnitude in dB

Phase in degrees

The SISO Tool Preferences Editor

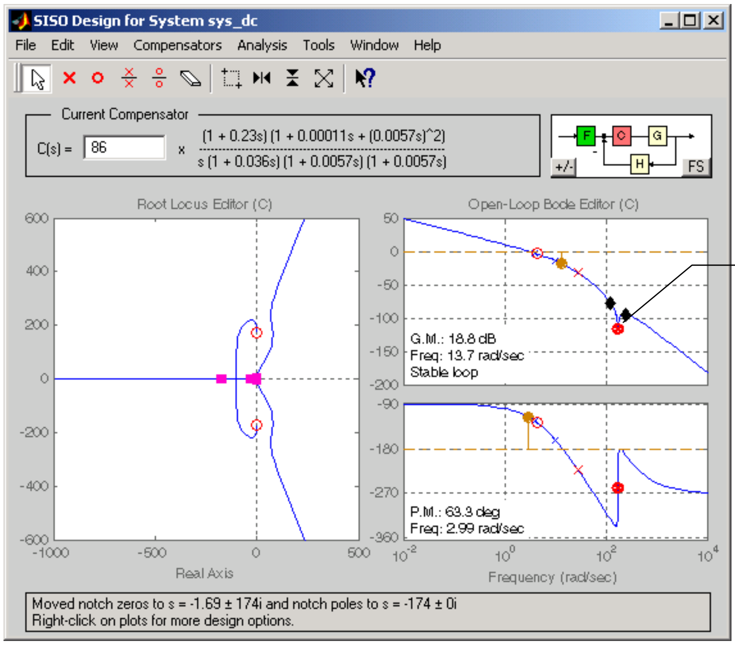
Use the menu options on the **Units** page to make the change. This unit change persists for the entire session.

For more information about property and preference setting, see “Customization” in the online documentation under the Control System Toolbox.

Adding a Notch Filter

If you know that you have disturbances to your system at a particular frequency, you can use a notch filter to attenuate the gain of the system at that frequency. To add a notch filter, select **Add Notch** from the right-click menu and place the filter at the frequency you want to attenuate. A black ‘x’ will appear next to the mouse arrow; place it at the frequency you want to attenuate.

This figure shows the result.


SISO Design for System sys_dc
File Edit View Compensators Analysis Tools Window Help
Current Compensator
C(s) = 86 x (1 + 0.23s) (1 + 0.00011s + (0.0057s)^2) / s (1 + 0.036s) (1 + 0.0057s) (1 + 0.0057s)
F C G H +/- FS
Root Locus Editor (C)
Real Axis
Open-Loop Bode Editor (C)
G.M.: 18.8 dB
Freq: 13.7 rad/sec
Stable loop
P.M.: 63.3 deg
Freq: 2.99 rad/sec
Frequency (rad/sec)
See Figure 4-15 for a close look at the notch filter parameters.

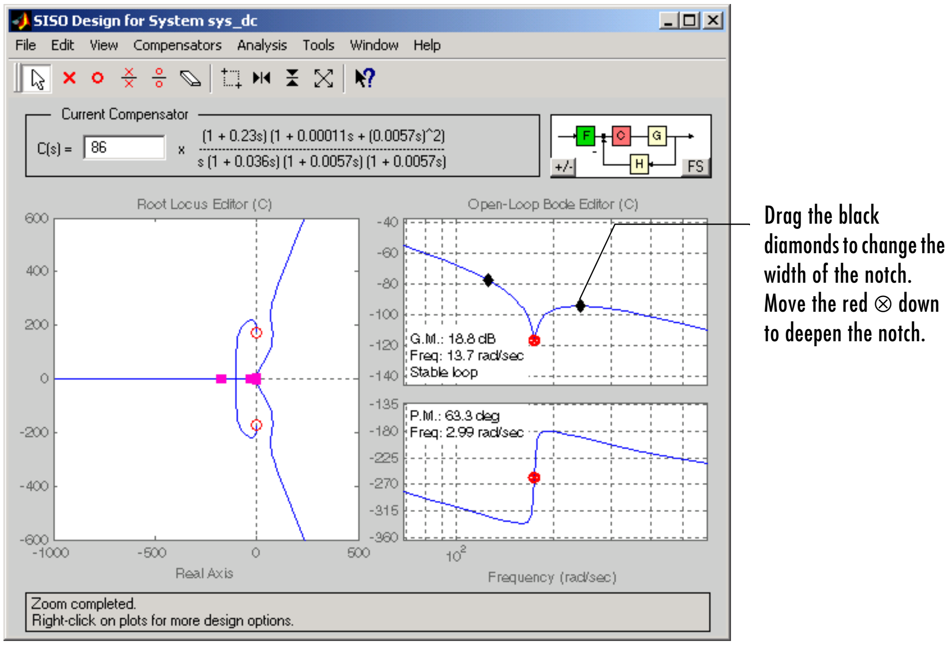
Moved notch zeros to s = -1.69 ± 174i and notch poles to s = -174 ± 0i
Right-click on plots for more design options.

The SISO Tool with a Notch Filter Added to the DC Motor Compensator

Note that to add the notch filter it was necessary to zoom out, since the notch frequency is at a higher frequency than the figure Final Design Parameters for the DC Motor Compensator on page 4-18 displayed.

To see the notch filter parameters in more detail, select **Zoom X-Y** in the right-click menu for the Bode magnitude plot. Left-click and drag your mouse to draw a box around the notch filter. When you release the mouse, the SISO Design Tool will zoom in on the selected region.

This figure zooms in on the notch filter to show the adjustable parameters.


SISO Design for System sys_dc
File Edit View Compensators Analysis Tools Window Help
Root Locus Editor (C)
Open-Loop Bode Editor (C)
Real Axis
Frequency (rad/sec)
Drag the black diamonds to change the width of the notch. Move the red ⊗ down to deepen the notch.

Current Compensator

C(s) = 86 x (1 + 0.23s) (1 + 0.00011s + (0.0057s)^2) / s (1 + 0.036s) (1 + 0.0057s) (1 + 0.0057s)

G.M.: 18.8 dB
Freq: 13.7 rad/sec
Stable loop

P.M.: 63.3 deg
Freq: 2.99 rad/sec

Zoom completed.
Right-click on plots for more design options.

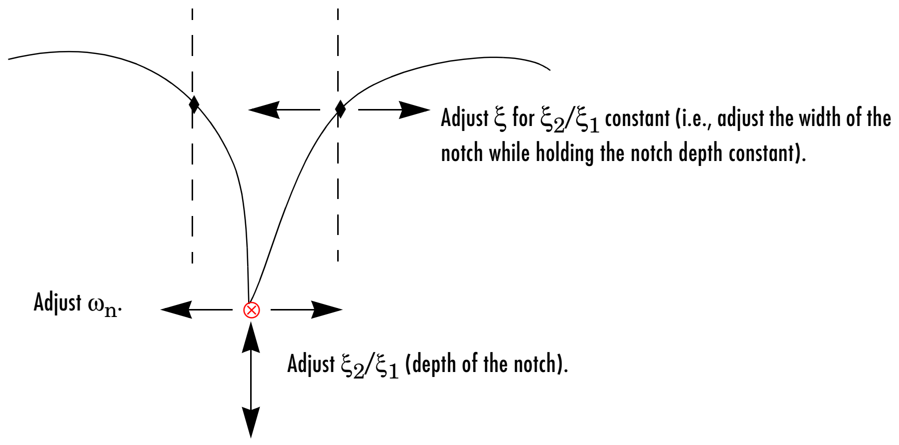
Manipulating Notch Filter Parameters

To understand how adjusting the notch filter parameters affects the filter, consider the notch filter transfer function.

$$\frac{s^2 + 2\xi_1\omega_n s + \omega_n^2}{s^2 + 2\xi_2\omega_n s + \omega_n^2}$$

The three adjustable parameters are ξ_1, ξ_2, and ω_n. The ratio of ξ_2/ξ_1 sets the depth of the notch, and ω_n is the natural frequency of the notch.

This diagram shows how moving the red ⊗ and black diamonds changes these parameters and hence the transfer function of the notch filter.


Adjust ξ for ξ_2/ξ_1 constant (i.e., adjust the width of the notch while holding the notch depth constant).
Adjust ω_n.
Adjust ξ_2/ξ_1 (depth of the notch).

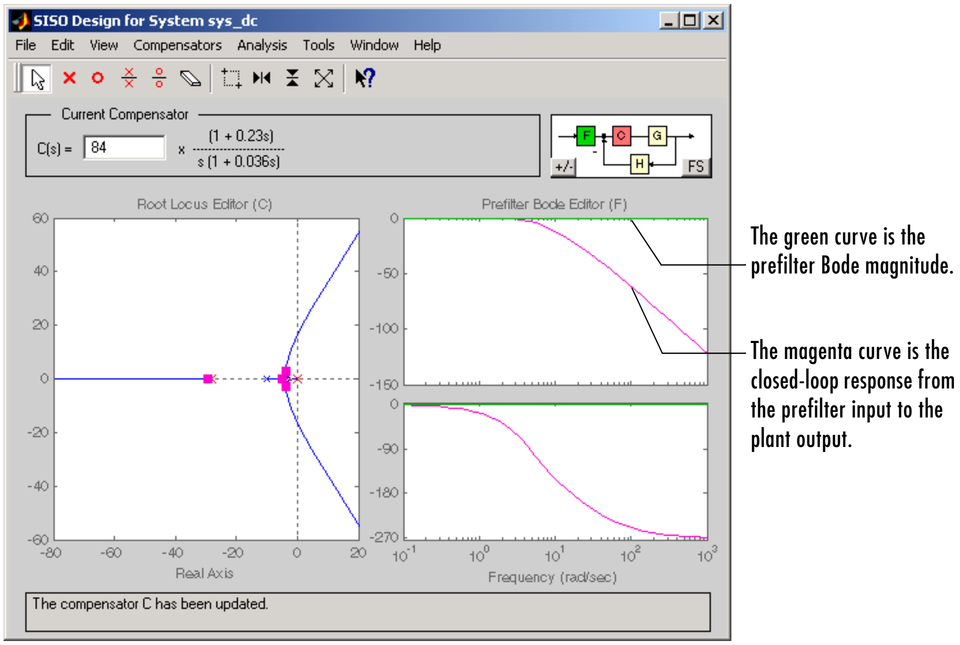
A Close Look at Notch Filter Parameters

Adding a Prefilter

You can use the SISO Design Tool to add and modify a prefilter to your design. Typical prefilter applications include:

- Achieving (near) feedforward tracking to reduce load on the feedback loop (when stability margins are poor)
- Filtering out high frequency content in the command (reference) signal to limit overshoot or to avoid exciting resonant modes of the plant

A common prefilter is a simple lowpass filter that reduces noise in the input signal. To do this, first open a Bode diagram for the prefilter by selecting **Prefilter Bode** from the **View** menu.


SISO Design for System sys_dc
File Edit View Compensators Analysis Tools Window Help
Current Compensator
C(s) = 84 x (1 + 0.23s) / s (1 + 0.036s)
F C G H
+/- FS
Root Locus Editor (C)
Real Axis
Prefilter Bode Editor (F)
Frequency (rad/sec)
The green curve is the prefilter Bode magnitude.
The magenta curve is the closed-loop response from the prefilter input to the plant output.

The compensator C has been updated.

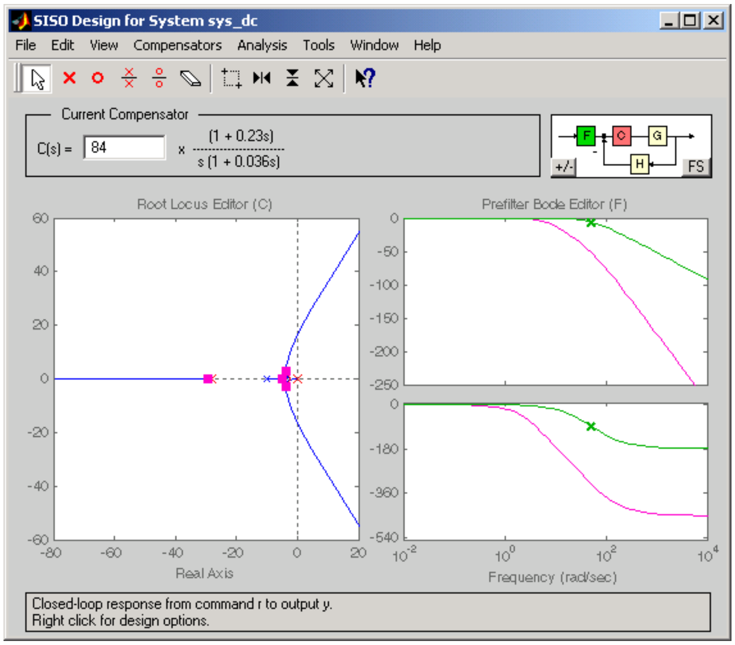
Opening the Prefilter Bode Diagram

For clarity, the above figure does not show the open-loop Bode diagram for the compensator (**C**). To remove the Bode diagram from the display, clear **Open-loop Bode** under the **View** menu.

If you haven't imported a prefilter, the default is a unity gain. You can add poles and zeros and adjust the gain using the same methods as you did when designing the compensator (C), including the following:

- The right-click menu
- The **Edit Compensator F** window
- The **Prefilter Bode** menu from **Edit**
- The toolbar

A quick way to create a lowpass roll-off filter is to add a pair of complex poles. To do this, first activate the grid for the prefilter Bode, then select **Add Pole/Zero** and **Complex Pole** from the prefilter Bode right-click menu. For this example, try to place the poles at about 50 rad/s. This figure shows the poles added to the prefilter Bode diagram.


SISO Design for System sys_dc
File Edit View Compensators Analysis Tools Window Help
Current Compensator
F C G H +/- FS
Root Locus Editor (C)
Real Axis
Prefilter Bode Editor (F)
Frequency (rad/sec)

C(s) = 84 x (1 + 0.23s) / s (1 + 0.036s)

Closed-loop response from command r to output y.
Right click for design options.

Adding a Complex Pair of Poles to the Prefilter Bode Diagram

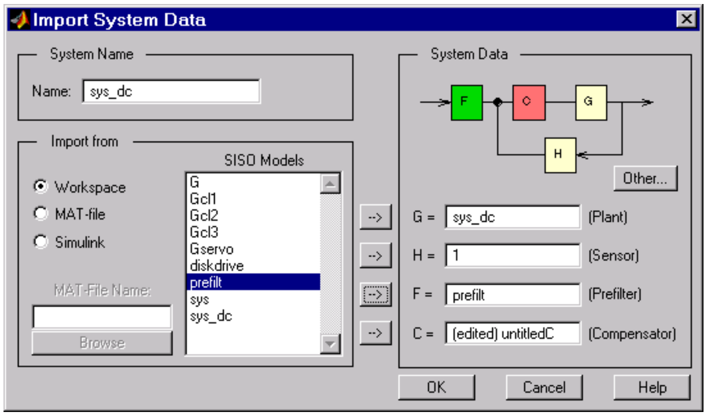
By default, the damping ratio of the complex pair is 1.0, which means that there are two real-valued poles at about -50 rad/s. The green curve, which represents the prefilter Bode response, shows the -3 dB point for the roll-off is at about 50 rad/s. The magenta curve, which represents the closed loop response from the prefilter to the plant output, shows that after the -3 dB point, the closed-loop gain rolls off at -40 dB/decade to provide some noise disturbance rejection.

Importing a Prefilter

An alternative approach is to design a prefilter using Control System Toolbox commands like `ss` or `tf` and importing the design directly into the prefilter. For example, to create the lowpass filter using `zpk`, try

```
prefilt=zpk([],[-35 + 35i, -35 - 35i],1)
```

and import `prefilt` by selecting **Import** from the **File** menu. This opens the **Import System Data** window.


Import System Data

System Name
Name: sys_dc

Import from
[✓ Workspace] MAT-file Simulink

SISO Models: G, Gcl1, Gcl2, Gcl3, Gservo, diskdrive, [✓ prefilt], sys, sys_dc

System Data
G = sys_dc (Plant)
H = 1 (Sensor)
F = prefilt (Prefilter)
C = (edited) untitledC (Compensator)

Importing a Prefilter

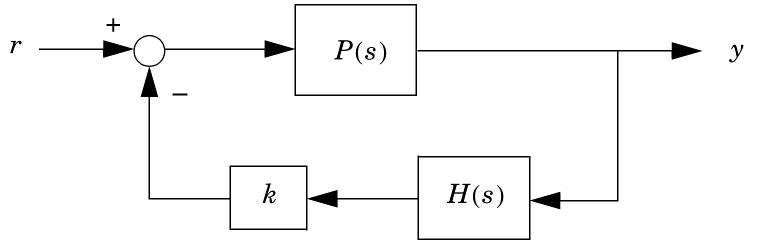
Select `prefilt` from the **SISO Models** list and click the arrow to the left of the **Prefilter** field to import the prefilter model. Once you've imported the prefilter model, you can modify it using the methods described above.

Root Locus Design

A common technique for meeting design criteria is root locus design. This approach involves iterating on a design by manipulating the compensator gain, poles, and zeros in the root locus diagram.

The root locus diagram shows the trajectories of the closed-loop poles of a feedback system as a single system parameter varies over a continuous range of values. Typically, the root locus method is used to tune the loop gain of a SISO control system by specifying a feedback gain the closed-loop pole locations.

Consider, for example, the tracking loop



where $P(s)$ is the plant, $H(s)$ is the sensor dynamics, and k is a scalar gain to be adjusted. The closed-loop poles are the roots of

$$q(s) = 1 + k\ P(s)H(s)$$

The root locus technique consists of plotting the closed-loop pole trajectories in the complex plane as k varies. You can use this plot to identify the gain value associated with a desired set of closed-loop poles.

The DC motor design example focused on the Bode diagram feature of the SISO Design Tool. Each of the design options available on the Bode diagram side of the tool have a counterpart on the root locus side. To demonstrate these techniques, this example presents an electrohydraulic servomechanism.

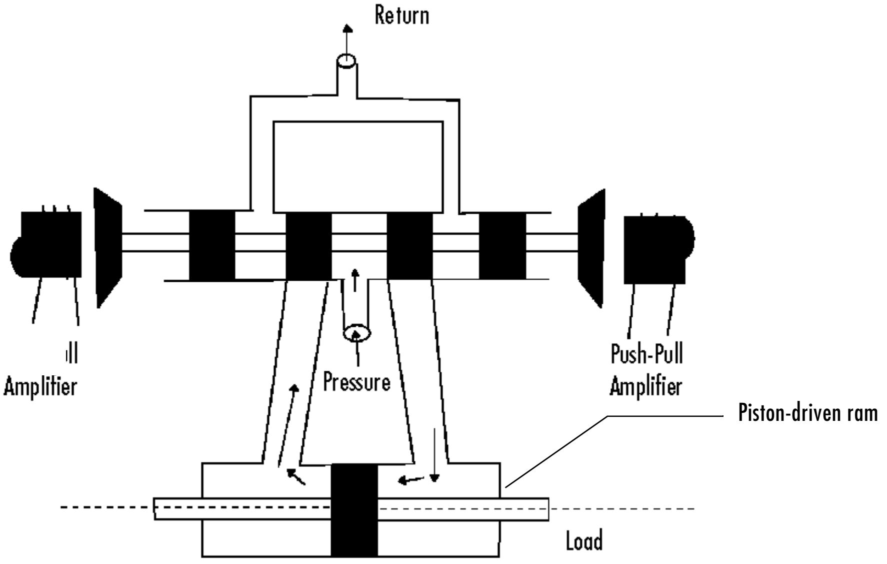
The SISO Design Tool's root locus and Bode diagram design tools provide complementary perspectives on the same design issues; each perspective offers insight into the design process. Since the SISO Design Tool displays both root locus and Bode diagrams, you can also choose to combine elements of both perspectives in making your design decisions.

Example: Electrohydraulic Servomechanism

A simple version of an electrohydraulic servomechanism model consists of

- A push-pull amplifier (a pair of electromagnets)
- A sliding spool in a vessel of high pressure hydraulic fluid
- Valve openings in the vessel to allow for fluid to flow
- A central chamber with a piston-driven ram to deliver force to a load
- A symmetrical fluid return vessel

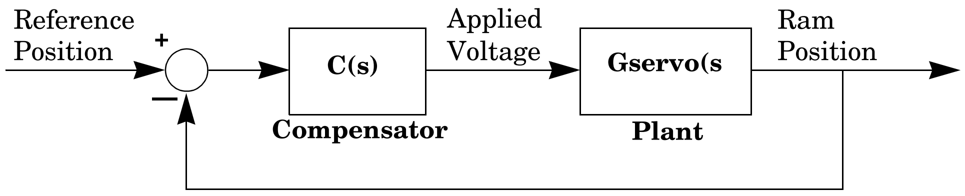
This figure shows a schematic of this servomechanism.



An Electrohydraulic Servomechanism

The force on the spool is proportional to the current in the electromagnet coil. As the spool moves, the valve opens, allowing the high-pressure hydraulic fluid to flow through the chamber. The moving fluid forces the piston to move in the opposite direction of the spool. *Control System Dynamics,* by R. N. Clark, (Cambridge University Press, 1996) derives linearized models for the electromagnetic amplifier, the valve spool dynamics, and the ram dynamics; it also provides a detailed description of this type of servomechanism.

If you want to use this servomechanism for position control, you can use the input voltage to the electromagnet to control the ram position. When measurements of the ram position are available, you can use feedback for the ram position control, as shown in the figure below.



Feedback Control Structure for an Electrohydraulic Servomechanism

Your task is to design the compensator, **C(s)**.

Plant Transfer Function

If you have not already done so, type

```
load ltiexamples
```

to load a collection of linear models that include `Gservo`, which is a linearized plant transfer function for the electrohydraulic position control mechanism. Typing `Gservo` at the MATLAB prompt displays the servomechanism (plant) transfer function.

```
Gservo

Zero/pole/gain from input "Voltage" to output "Ram position":
        40000000
----------------------------
s (s+250) (s^2 + 40s + 9e004)
```

Design Specifications

For this example, you want to design a controller so that the step response of the closed-loop system meets the following specifications:

- The 2% settling time is less than 0.05 second.
- The maximum overshoot is less than 5%.

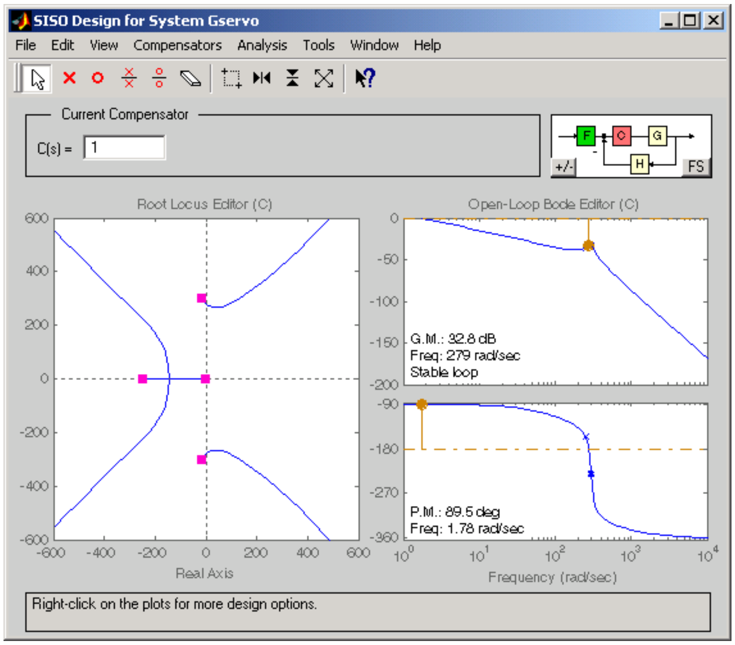
The remainder of this section discusses how to use the SISO Design Tool to design a controller to meet these specifications.

Opening the SISO Tool

Open the SISO Design Tool and import the model by typing

```
sisotool(Gservo)
```

at the MATLAB prompt. This opens the SISO Design Tool with the servomechanism plant imported.


SISO Design for System Gservo
File Edit View Compensators Analysis Tools Window Help
Current Compensator
F
C
G
H
+/-
FS
Root Locus Editor (C)
Open-Loop Bode Editor (C)
600
400
200
0
-200
-400
-600
Real Axis
Frequency (rad/sec)

C(s) = 1

G.M.: 32.8 dB
Freq: 279 rad/sec
Stable loop

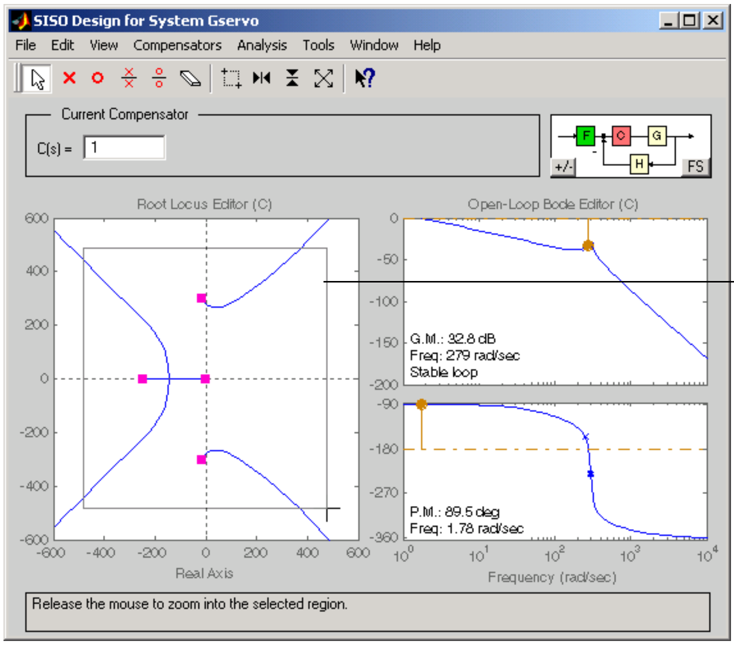
P.M.: 89.5 deg
Freq: 1.78 rad/sec

Right-click on the plots for more design options.

SISO Design Tool Showing the Root Locus and Bode Plots for the Electrohydraulic Servomechanism Plant

Zooming

Using the right-click menu in the root locus, select **X-Y** under **Zoom**. Hold down the mouse's left button and drag the mouse to select a region for zooming. For this example, reduce the root locus region to about -500 to 500 in both the *x*- and *y*-axes. This figure illustrates the zooming in process.


SISO Design for System Gservo
File Edit View Compensators Analysis Tools Window Help
Current Compensator
C(s) = 1
F
C
G
H
+/-
FS
Root Locus Editor (C)
Real Axis
Open-Loop Bode Editor (C)
Frequency (rad/sec)

G.M.: 32.8 dB
Freq: 279 rad/sec
Stable loop

P.M.: 89.5 deg
Freq: 1.78 rad/sec

Release the mouse to zoom into the selected region.

Hold down your mouse's left button to select a rectangular region for zooming in. When you release the button, the SISO Design Tool replots the root locus with the new axis boundaries.

To undo the zoom, select **Zoom Out** in the right-click menu.

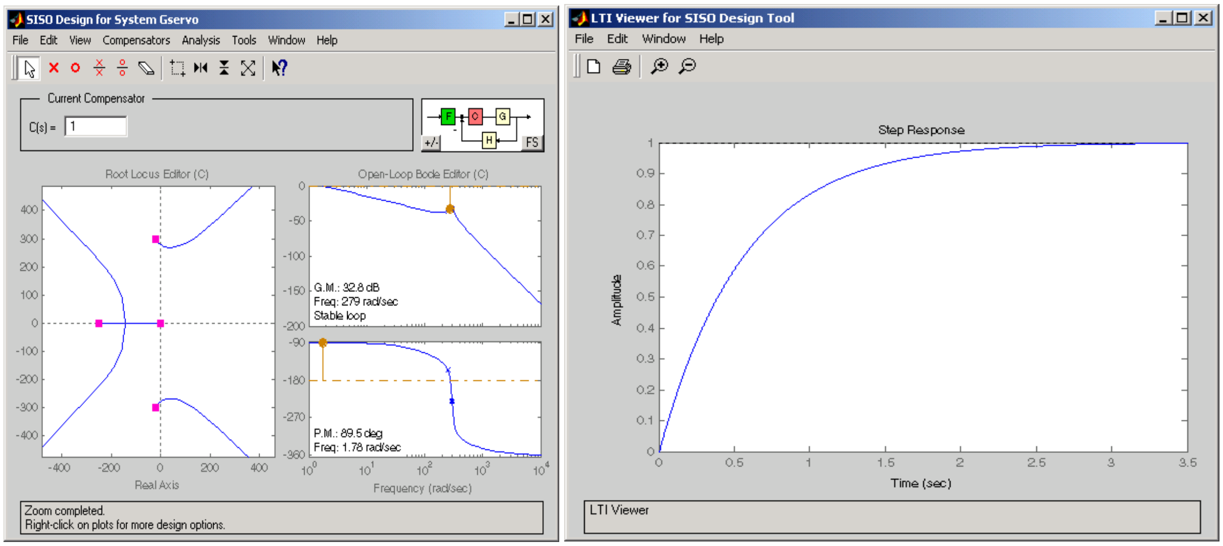
Zooming in on a Region in the Root Locus Plot

Alternatively, you use the zoom icons on the toolbar:

- — Zoom in X-Y
- — Zoom in X
- — Zoom in Y
- — Zoom out

As in the DC motor example, open an LTI Viewer by selecting **Other Response** under **Analysis** in the menu bar. Click **OK** on the **Response Plot Setup** window; the default setting is a closed-loop step response from reference signal *r* to output signal *y*. For more information about the **Response Plot Setup** window, see “Customizing Loop Responses” in the online documentation under the “Tool and Viewer Reference.”

You now should have two windows, the SISO Design Tool and the associated LTI Viewer side by side.



Zoom completed.
Right-click on plots for more design options.

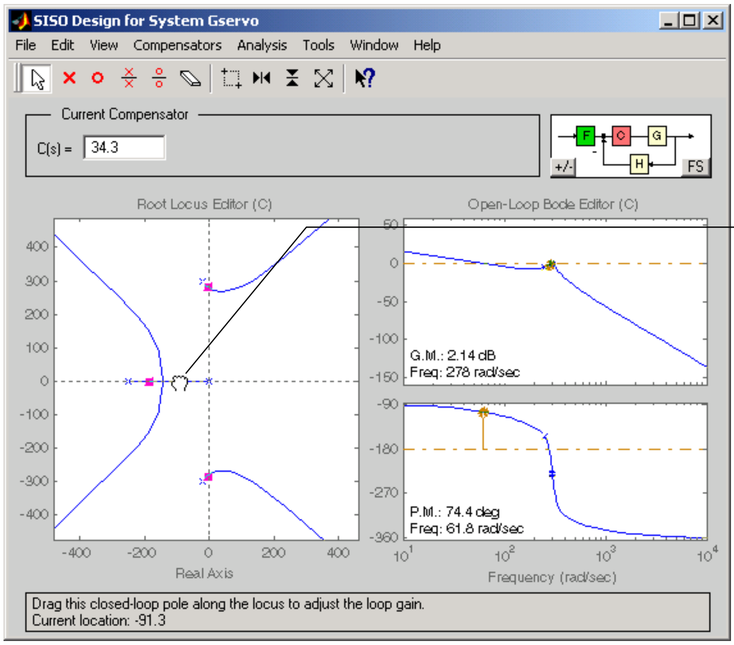
LTI Viewer

SISO Design Tool and Associated LTI Viewer for the Electrohydraulic Servomechanism

The step response plot shows that the rise time is on the order of 2 seconds, which is much too slow given the system requirements. The following sections describe how to use frequency design techniques in the SISO Design Tool to design a compensator that meets the requirements specified in “Design Specifications” on page 4-29.

Changing the Compensator Gain

The simplest thing to do is change the compensator gain, which by default is unity. You can change the gain by grabbing the red squares on the root locus plot and moving them along the curve. This figure shows the procedure.



Drag this closed-loop pole along the locus to adjust the loop gain.
Current location: -91.3

Move the red squares to change the compensator gain. The SISO Design Tool calculates the compensator gain **C(s)** and displays it in the **Current Compensator** panel.

Changing the Compensator Gain in the Root Locus Plot

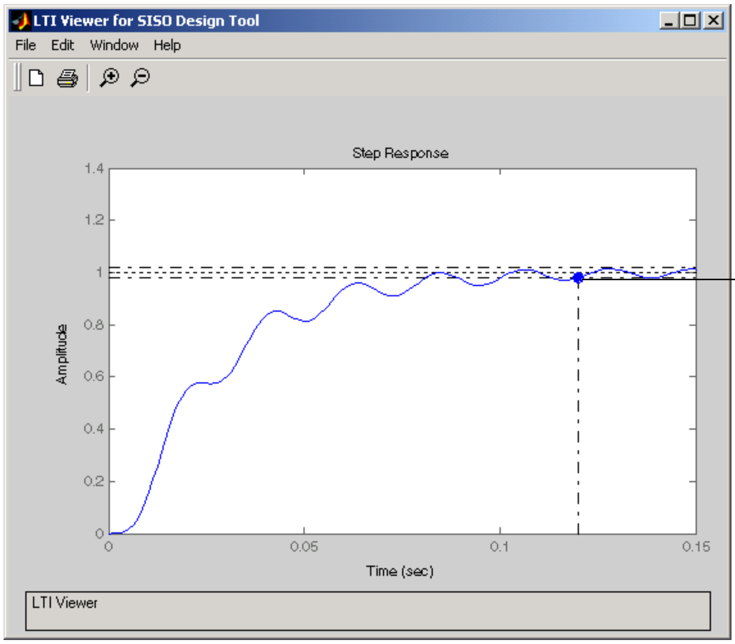
Experiment with different gains and view the closed-loop response in the associated LTI Viewer.

Alternatively, you can change the compensator gain by entering values into the **C(s)** field in the **Current Compensator** panel.

Closed-Loop Response

Change the gain to 20 by editing the text box next to **Gain**, and pressing the **Enter** key. Notice that the locations of the closed-loop poles on the root locus are recalculated for the new gain set point.

This figure shows the associated closed-loop step response for the gain of 20.


LTI Viewer for SISO Design Tool
File Edit Window Help
Step Response
Amplitude
Time (sec)
LTI Viewer
Select **Settling Time** under **Characteristics** in the LTI Viewer's right-click menu to display the settling for this response.

Step Response with the Settling Time for C(s) = 20

This closed-loop response does not meet the desired settling time requirement (0.05 seconds or less) and exhibits unwanted ringing. The next section shows how to design a compensator so that you meet the required specifications.

Adding Poles and Zeros to the Compensator

You may have noticed that increasing the gain makes the system underdamped. Further increases force the system into instability, so meeting the design requirements with only a gain in the compensator is not possible.

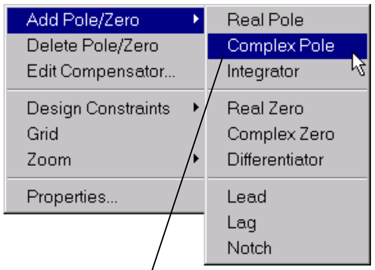
There are three sets of parameters that specify the compensator: poles, zeros, and gain. Once you have selected the gain, you can add poles or zeros to the compensator.

Adding Poles to the Compensator

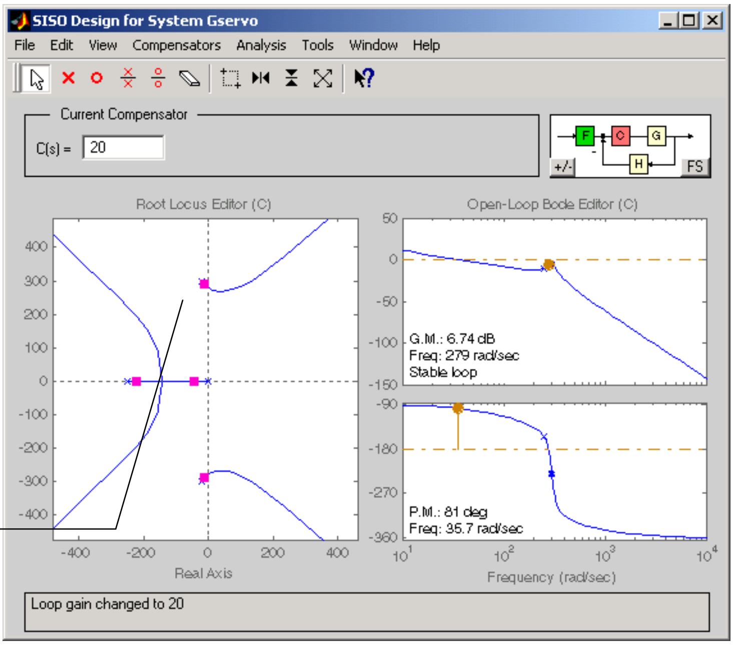
Try adding a complex conjugate compensator pole pair on the root locus plot:

1. Open the right-click menu and select **Add Pole/Zero** and then **Complex Pole**.
2. Click on the root locus plot region where you would like to add one of the complex poles.

This figure shows these two steps.



Select **Add/Complex Pole** in the right-click menu.



C(s) = 20

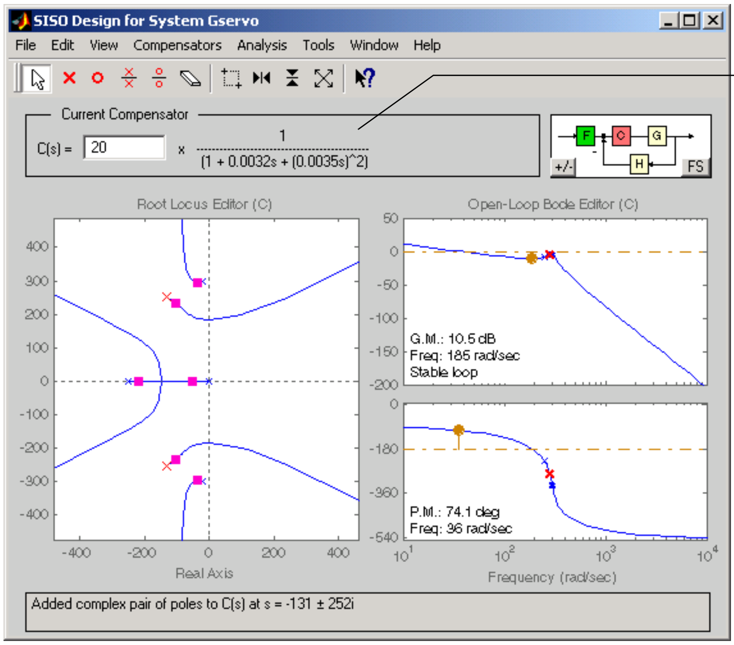
Loop gain changed to 20

Add the poles by placing the ‘x’ on the Bode magnitude plot. The SISO Design Tool places the poles at the location that you mark.

Adding a Complex Pair of Poles to the Compensator

Try placing the ‘x’ somewhere to the left of the complex pole pair near the imaginary axis (the figure above shows a good spot). Once you have added the complex pair of poles, the LTI Viewer response plots change and both the root locus and Bode plots display the new poles. This figure shows the SISO Design

Tool with the new poles added. For clarity, you may want to zoom out further, as was done here.



Added complex pair of poles to C(s) at s = -131 ± 252i

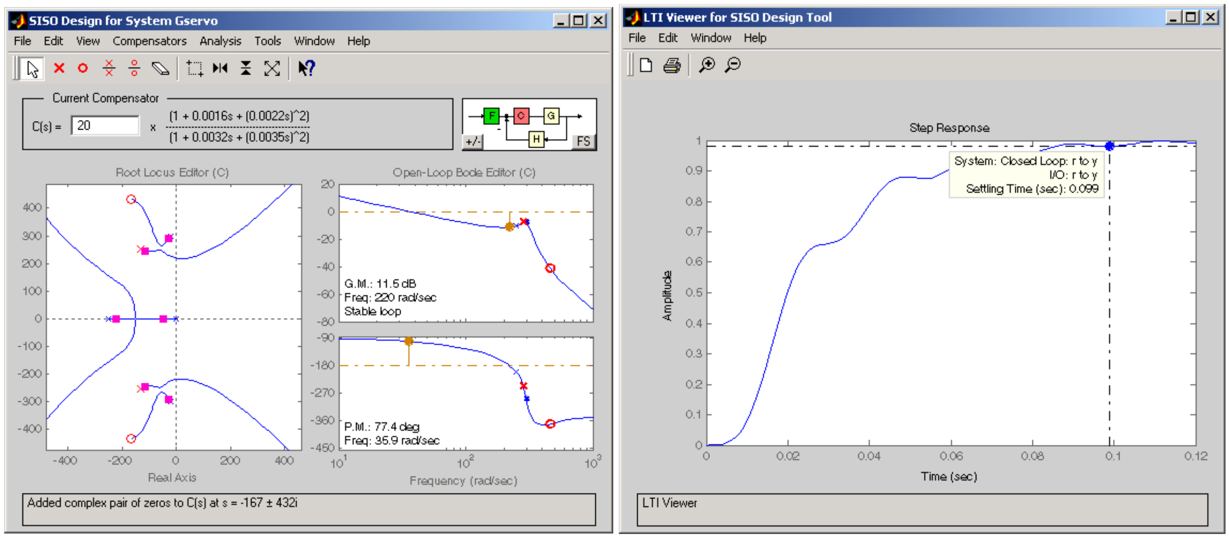
The **Current Compensator** displays the new complex pair of poles. If you see NumC or DenC, widen the GUI to see the full transfer function.

The SISO Design Tool displays the new poles as a pair of red x's on the root locus plot.

The Result of Adding a Complex Pair of Poles to the Compensator

Adding Zeros to the Compensator

The procedure for adding zeros to the compensator is exactly the same. Try adding a pair of complex zeros just to the left of complex closed-loop poles you just added to the compensator. This figure shows the results.



C(s) = 20 x (1 + 0.0016s + (0.0022s)^2) / (1 + 0.0032s + (0.0035s)^2)

Added complex pair of zeros to C(s) at s = -167 ± 432i

Electrohydraulic Servomechanism Example with Complex Zeros Added

If your step response is unstable, lower the gain by grabbing a red box in the right-hand plane and moving it into the left-half plane. In this example, the resulting step response is stable, but it still doesn't meet the design criteria since the 2% settling time is greater than 0.05 second.

As you can see, the compensator design process can involve some trial and error. You can try dragging the compensator poles, compensator zeros, or the closed-loop poles around the root locus until you meet the design criteria

The next section shows you how to place poles and zeros by specifying their numerical values. It also presents a solution that meets the design specifications for the servomechanism example.

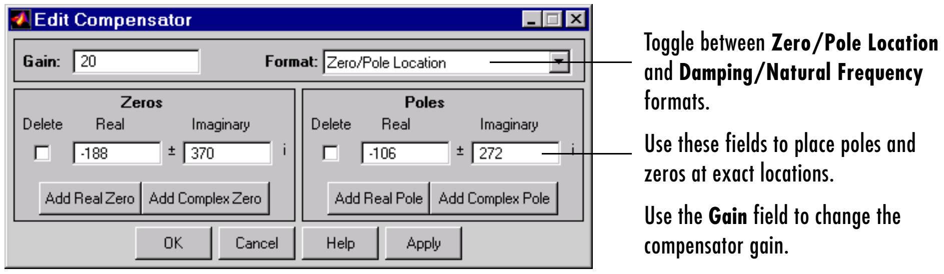
Editing Compensator Pole and Zero Locations

A quick way to change poles and zeros is simply to grab them with your mouse and move them around the root locus plot region. If you want to specify precise numerical values, however, you should use the **Edit Compensator** window to change the gain value and the pole and zero locations of your compensator.

There are three ways to open the **Edit Compensator** window from the SISO Design Tool:

- Select **Edit Compensator** under the **Edit** menu on the menu bar.
- Select **Edit Compensator** in the right-click menu. This option is available in both the root locus and Bode plot right-click menus.
- Double-click your mouse in the **Current Compensator** panel.

Whichever method you choose, the following window appears.



Gain: 20

	Real		Imaginary	
Zeros	-188	±	370	i
Poles	-106	±	272	i

Toggle between **Zero/Pole Location** and **Damping/Natural Frequency** formats.

Use these fields to place poles and zeros at exact locations.

Use the **Gain** field to change the compensator gain.

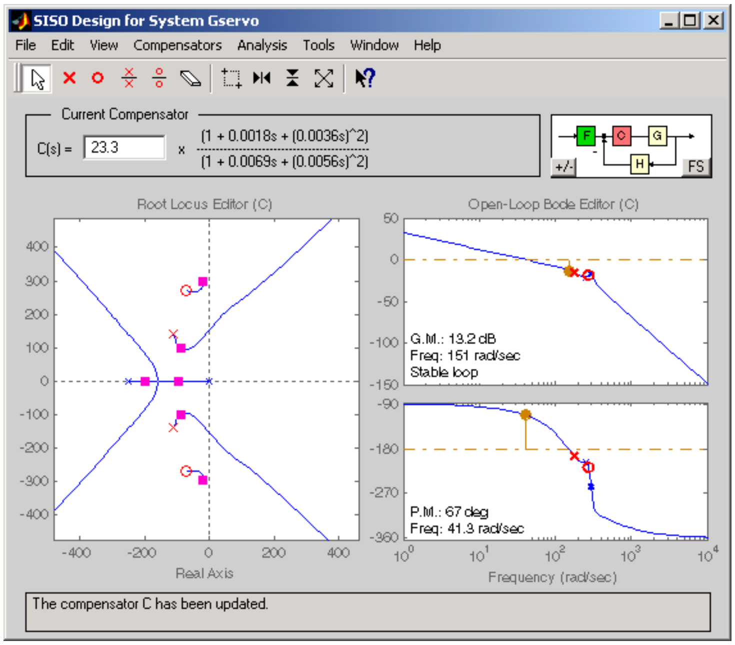
Use the Edit Compensator Window to Add, Delete and Move Compensator Poles and Zeros

You can use the **Edit Compensator** window to

- Edit the compensator gain.
- Edit the locations of compensator poles and zeros.
- Add compensator poles and zeros.
- Delete compensator poles and zeros.

For this example, edit the poles to be at $-110 \pm 140i$ and the zeros at $-70 \pm 270i$. Set the compensator gain to 23.3.

Your SISO Design Tool now looks like this.


SISO Design for System Gservo
File Edit View Compensators Analysis Tools Window Help
Current Compensator
F C G H
+/-
FS
Root Locus Editor (C)
Real Axis
Open-Loop Bode Editor (C)
Frequency (rad/sec)

C(s) = 23.3 x (1 + 0.0018s + (0.0036s)^2) / (1 + 0.0069s + (0.0056s)^2)

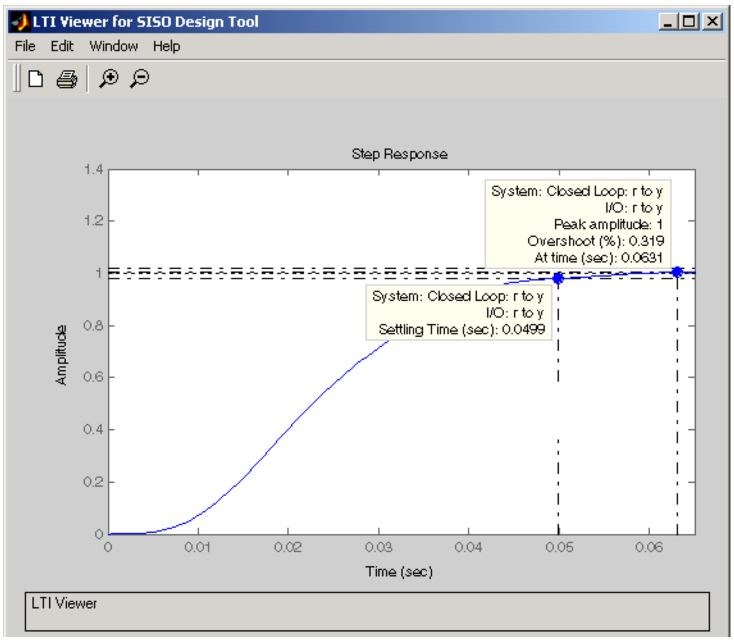
G.M.: 13.2 dB
Freq: 151 rad/sec
Stable loop

P.M.: 67 deg
Freq: 41.3 rad/sec

The compensator C has been updated.

SISO Design Tool with the Final Values for the Electrohydraulic Servomechanism Design Example

To see that this design meets the design requirements, take a look at the step response of the closed-loop system.

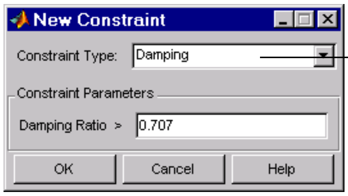


Closed-Loop Step Response for the Final Design of the Electrohydraulic Servomechanism Example

The step response looks good. As you can see, the settling time is less than 0.05 second, and the overshoot is less than 5%. You have met the design specifications.

Viewing Damping Ratios

The SISO Design Tool provides design constraints that can make it easier to meet design specifications. If you want to place, for example, a pair of complex poles on your diagram at a particular damping ratio, select **Design Constraints** and then **New** from the right-click menu in the root locus. This opens the **Design Constraints** editor.



Constraint Type: Damping

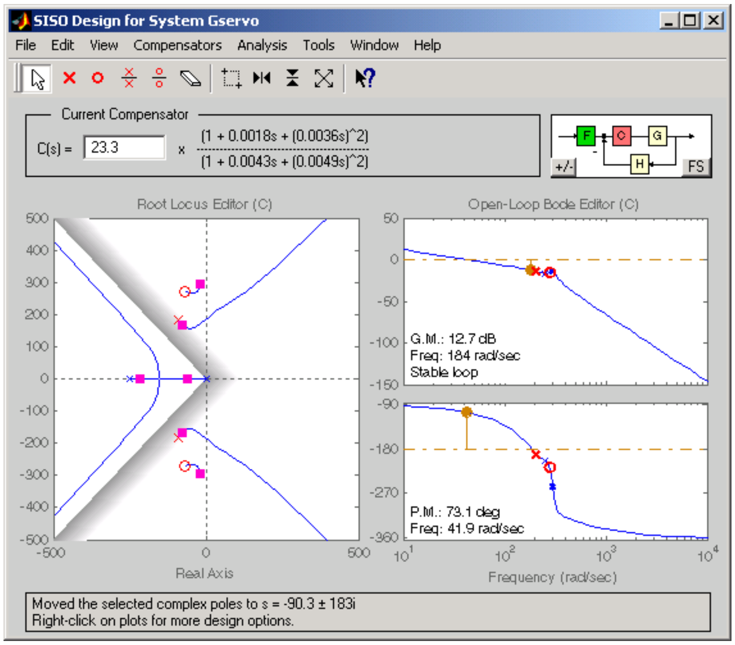
Constraint Parameters

Damping Ratio > 0.707

Select Damping from this pull-down menu. The default value 0.707 sets lines on the root locus corresponding to a 0.707 damping ratio.

The New Constraint Editor

Applying damping ratios to the root locus plot results in a pair of shaded rays at the desired slope, as this figure shows.


SISO Design for System Gservo
File Edit View Compensators Analysis Tools Window Help
Current Compensator
FS
+/-
Root Locus Editor (C)
Open-Loop Bode Editor (C)
Real Axis
Frequency (rad/sec)

C(s) = 23.3 x (1 + 0.0018s + (0.0036s)^2) / (1 + 0.0043s + (0.0049s)^2)

G.M.: 12.7 dB
Freq: 184 rad/sec
Stable loop

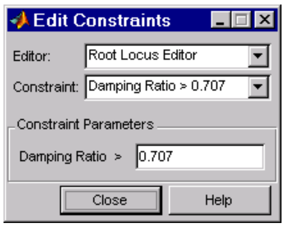
P.M.: 73.1 deg
Freq: 41.9 rad/sec

Moved the selected complex poles to s = -90.3 ± 183i
Right-click on plots for more design options.

Root Locus Displaying 0.707 Damping Ratio Lines

Try moving the complex pair of poles you've added to the design so that they are on the 0.707 damping ratio line. You can experiment with different damping ratios to see the effect on the design.

If you want to change the damping ratio, select **Design Constraint** and then **Edit** from the right-click menu. This opens the **Edit Constraints** window.



Editor: Root Locus Editor
Constraint: Damping Ratio > 0.707
Constraint Parameters
Damping Ratio > 0.707

The Edit Constraints Window

Specify the new damping ratio in this window.

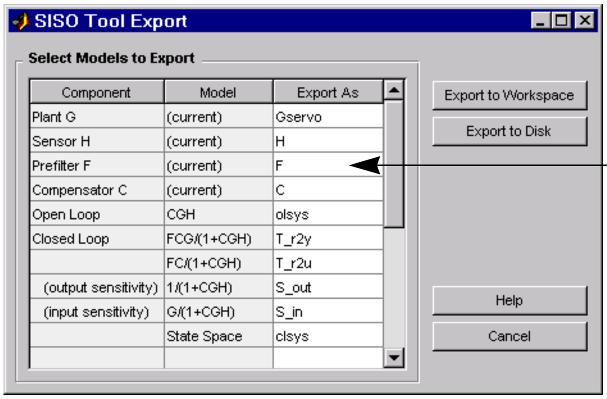
An alternate way to adjust a constraint is to left-click on the constraint itself to select it. Two black squares appear on the constraint when it is selected. You can then drag it with your mouse anywhere in the plot region.

If you want to add a different set of constraints, for example, a settling time constraint, reopen the **New Constraint** window and choose settling time from the pull-down menu. You can have multiple types of design constraints in one plot, or more than one instance of any type.

The types of constraints available depend on which view you use for your design. See "Design Constraints" in the Control System Toolbox online documentation for a description of all the constraint types available in the SISO Design Tool.

Exporting the Compensator and Models

Now that you have successfully designed your compensator, you may want to save your design parameters for future implementation. You can do this by selecting **Export** from the **File** menu on the SISO Design Tool. The window shown below opens.



Component	Model	Export As
Plant G	(current)	Gservo
Sensor H	(current)	H
Prefilter F	(current)	F
Compensator C	(current)	C
Open Loop	CGH	olsys
Closed Loop	FCG/(1+CGH)	T_r2y
	FC/(1+CGH)	T_r2u
(output sensitivity)	1/(1+CGH)	S_out
(input sensitivity)	G/(1+CGH)	S_in
	State Space	clsys

SISO Tool Export Window

The variables listed in the **Export As** column are either previously named by you (in the **Import System Data** window) or have default names. To export your compensator to the workspace,

1 Select Compensator C in the **Component** column. If you want to change the export name, double-click in the cell for Compensator C.

2 Click on the **Export to Workspace** button.

If you go to the MATLAB prompt and type

```
who
```

the compensator is now in the workspace, in the variable named `C`.

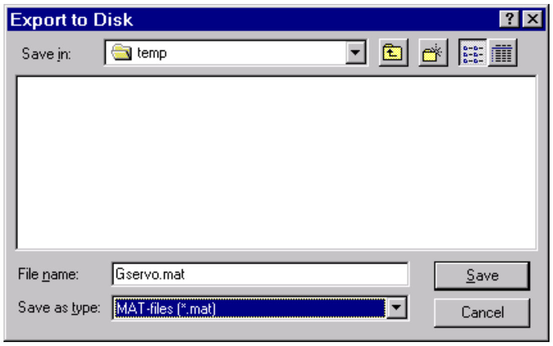
Type

```
C
```

to see that this variable is stored in `zpk` format.

To select multiple components, use the **Shift** key if they are all adjacent and the **Ctrl** key if they are not.

Selecting **Export to Disk** opens the window shown below.



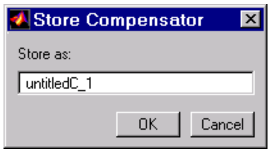
File name: Gservo.mat

Save as type: MAT-files (*.mat)

You can save your models as MAT-files in any directory you want. The default name for the MAT-file is the name of your original model; you can change the name to anything you want. If you save multiple components, they are stored in a single MAT-file.

Storing and Retrieving Intermediate Designs

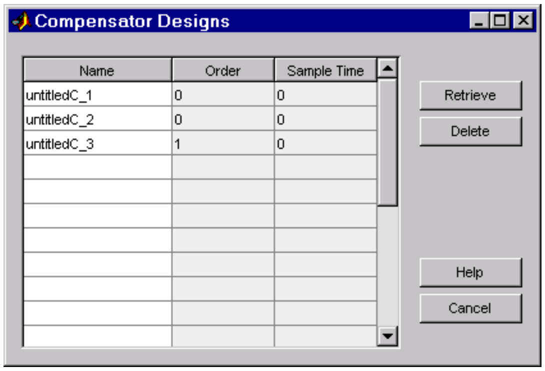
You can store and retrieve intermediate compensators while you iterate on your compensator design. To store intermediate designs, select **Store** under **Compensator** in the menu bar of the SISO Design Tool. This window opens.



Store as: untitledC_1

The default name is `UntitledC_1`; the suffix increments when you store additional compensators. You can rename the designs by editing the **Store as** field.

To retrieve intermediate designs, select **Retrieve** under the **Compensator** menu. This opens the **Compensator Designs** window.



Name	Order	Sample Time
untitledC_1	0	0
untitledC_2	0	0
untitledC_3	1	0

The Compensator Designs Window

To retrieve a design, select it from the list of names and click **Retrieve**. The SISO Design Tool automatically reverts to the selected compensator design. Note that you can rename the stored compensator by editing the name in the selected **Name** cell.

The **Compensator Designs** window also lists the order of each compensator design, and, if the compensator is digital, the sample time. The default sample time value for continuous-time compensators is 0.

You can delete an intermediate design by selecting it and clicking the **Delete** button.

Nichols Plot Design

An alternative method for designing compensators is to use the Nichols plot, which combines gain and phase information in a single plot. The combination of the two is useful when you are designing to gain and phase margin specifications.

You can design compensators with the SISO Design Tool by using Nichols plot techniques. This section repeats the DC motor compensator design presented in "Example: DC Motor" on page 4-8, only this time the focus is on Nichols plot techniques. The design strategy, however, is the same:

1. Adjust the compensator gain to improve the rise time.
2. Add an integrator to eliminate steady-state error.
3. Add a lead network to further improve the rise time while minimizing overshoot.

DC Motor Example

From "SISO Example: the DC Motor" on page 2-4, the transfer function of the DC motor is

```
Transfer function:
       1.5
------------------
s^2 + 14 s + 40.02
```

This example uses the design criteria specified in "Design Specifications" on page 4-29:

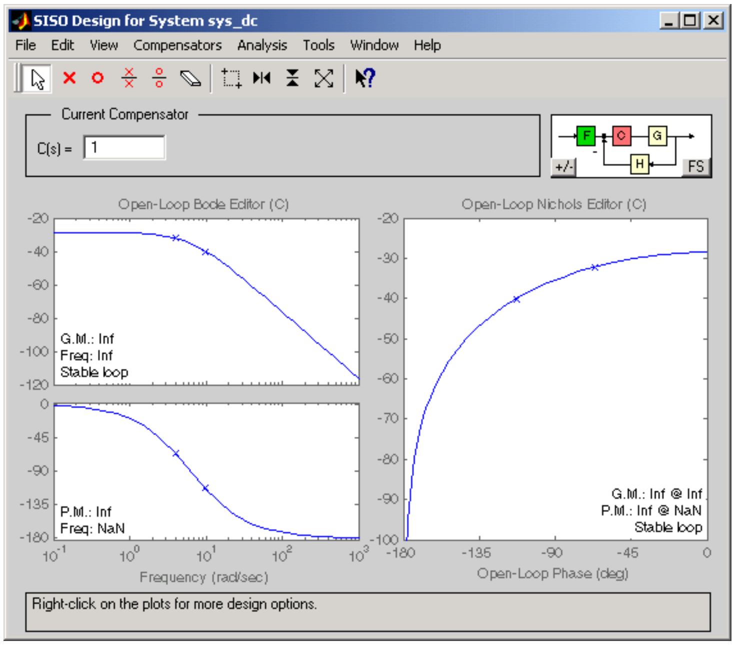
- Rise time of less than 0.5 second
- Steady-state error of less than 5%
- Overshoot of less than 10%
- Gain margin greater than 20 dB
- Phase margin greater than 40 degrees

Opening a Nichols Plot

To open the SISO Design Tool with a Bode diagram and a Nichols plot, use these commands.

```
load ltiexamples
sisotool({'bode','nichols'},sys_dc)
```

This figure opens.

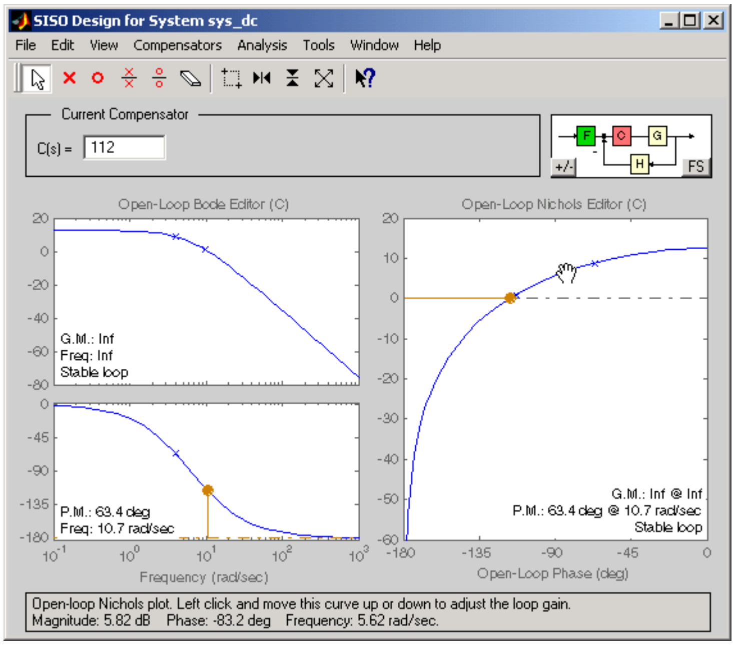


Right-click on the plots for more design options.

SISO Design Tool with a Bode Diagram and a Nichols Plot

Adjusting the Compensator Gain

You can adjust the compensator gain by moving the Nichols curve up and down with your mouse. To do this, place your mouse over the curve. The cursor turns into a hand. Left-click and move the curve up to increase the gain.


SISO Design for System sys_dc
File Edit View Compensators Analysis Tools Window Help
Current Compensator
F
C
G
H
+/-
FS
Open-Loop Bode Editor (C)
Open-Loop Nichols Editor (C)
Frequency (rad/sec)
Open-Loop Phase (deg)

C(s) = 112

G.M.: Inf
Freq: Inf
Stable loop

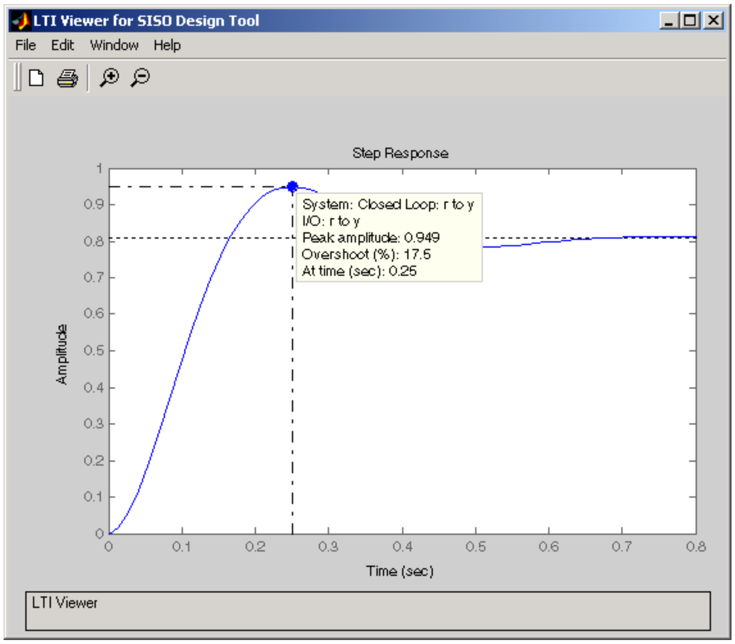
P.M.: 63.4 deg
Freq: 10.7 rad/sec

G.M.: Inf @ Inf
P.M.: 63.4 deg @ 10.7 rad/sec
Stable loop

Open-loop Nichols plot. Left click and move this curve up or down to adjust the loop gain.
Magnitude: 5.82 dB Phase: -83.2 deg Frequency: 5.62 rad/sec.

Adjusting the Compensator Gain by Moving the Nichols Curve

In this example, the new gain is 112. Select **Other Responses** from the **Analysis** menu to open the **Response Plot Setup** window. Click **OK** to open a linked LTI Viewer with the closed-loop response from reference signal *r* to output signal *y*.

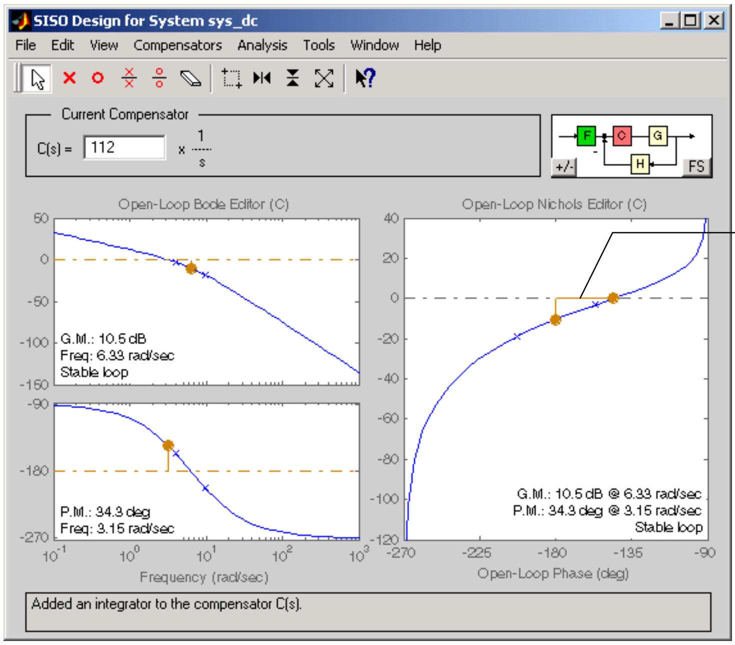


LTI Viewer Step Response for Compensator Gain = 112

The rise time is quite fast, about 0.15 second, but the overshoot is 18.4% and the steady-state is about 0.82.

Adding an Integrator

One approach to eliminating the steady-state error is to add an integrator. You can do this by selecting **Add Pole/Zero** and then **Integrator** from the right-click menu.

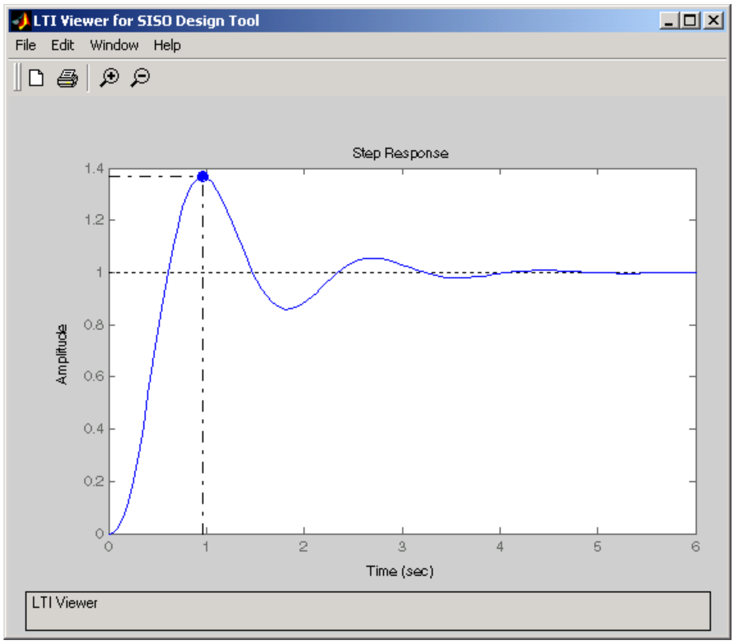

SISO Design for System sys_dc
File Edit View Compensators Analysis Tools Window Help
Current Compensator
C(s) = 112 x 1/s
F C G H +/- FS
Open-Loop Bode Editor (C)
Open-Loop Nichols Editor (C)
G.M.: 10.5 dB
Freq: 6.33 rad/sec
Stable loop
P.M.: 34.3 deg
Freq: 3.15 rad/sec
Frequency (rad/sec)
G.M.: 10.5 dB @ 6.33 rad/sec
P.M.: 34.3 deg @ 3.15 rad/sec
Stable loop
Open-Loop Phase (deg)
These lines indicate the gain and phase margins.

Added an integrator to the compensator C(s).

Adding an Integrator

Adding an integrator changes the gain margin from infinity to 10.5 dB. Since the gain and phase margins are now both finite, the Nichols plot displays a vertical line for the gain margin and a horizontal line for the phase margin.

The linked LTI Viewer automatically updates to show the new response.



Step Response for a Compensator Consisting of a Gain and an Integrator

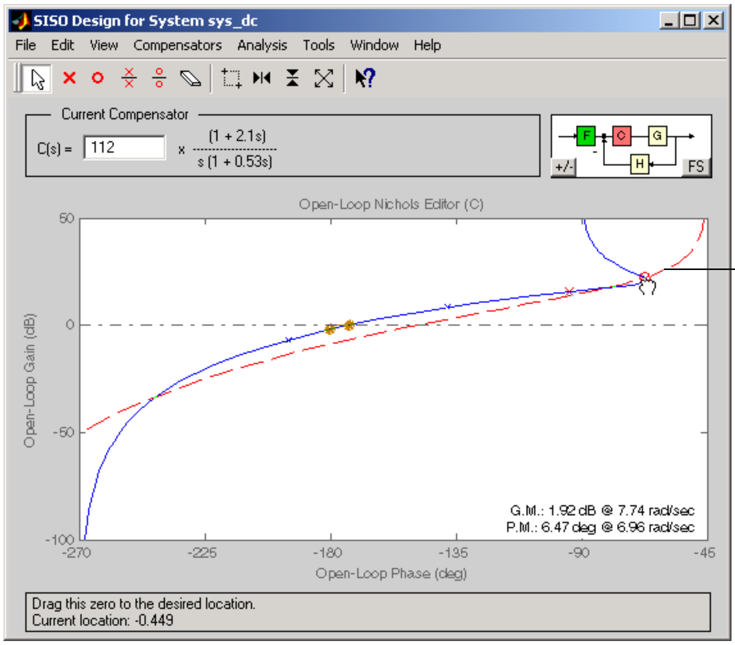
The steady-state value is now 1, which means the steady-state error has been eliminated, but the overshoot is 34% and the rise time is about 0.7 second. You must do more work to create a good design.

Adding a Lead Network

Improving the rise time requires that you increase the compensator gain, but increasing the gain further deteriorates the gain and phase margins while increasing the overshoot. You need to add a lead network to selectively raise the gain about the frequency crossover.

To add a lead network, select **Add Pole/Zero** and then **Lead** from the right-click menu. Your cursor turns into a red ‘x’. Left click along the Nichols curve to add the lead network. To move the lead network along the curve, left-click on the pole or zero and drag.

This figure shows the process of moving the lead network pole using the mouse (the open-loop Bode diagram has been removed for clarity).

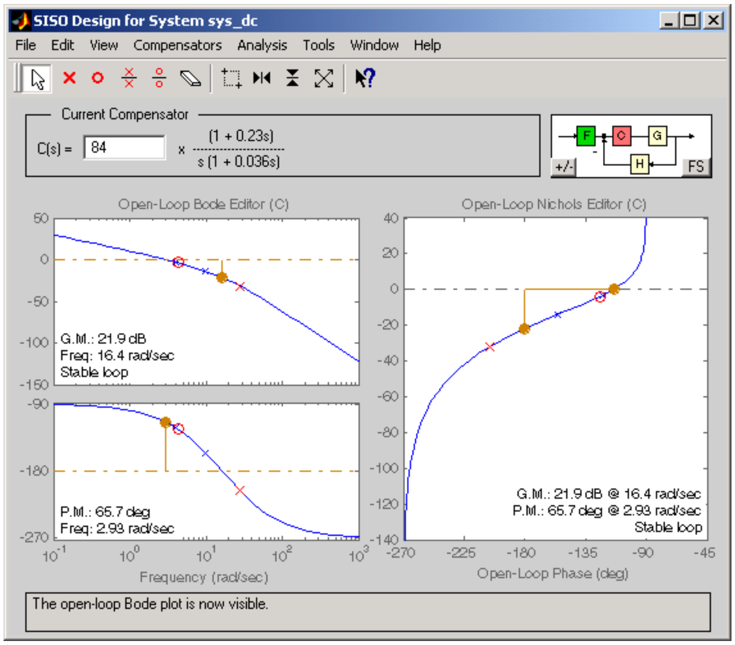

SISO Design for System sys_dc
File Edit View Compensators Analysis Tools Window Help
Current Compensator
C(s) = 112 x (1 + 2.1s) / s (1 + 0.53s)
F C G H +/- FS
Open-Loop Nichols Editor (C)
Open-Loop Gain (dB)
50
0
-50
-100
-270
-225
-180
-135
-90
-45
Open-Loop Phase (deg)
G.M.: 1.92 dB @ 7.74 rad/sec
P.M.: 6.47 deg @ 6.96 rad/sec
This dashed line shows the location of possible pole positions.

```
Drag this zero to the desired location.
Current location: -0.449
```

Moving the Lead Network's Pole

You can track the pole's movement in the Status Panel at the bottom of the SISO Design Tool. The Status Panel tells you the current location of the pole.

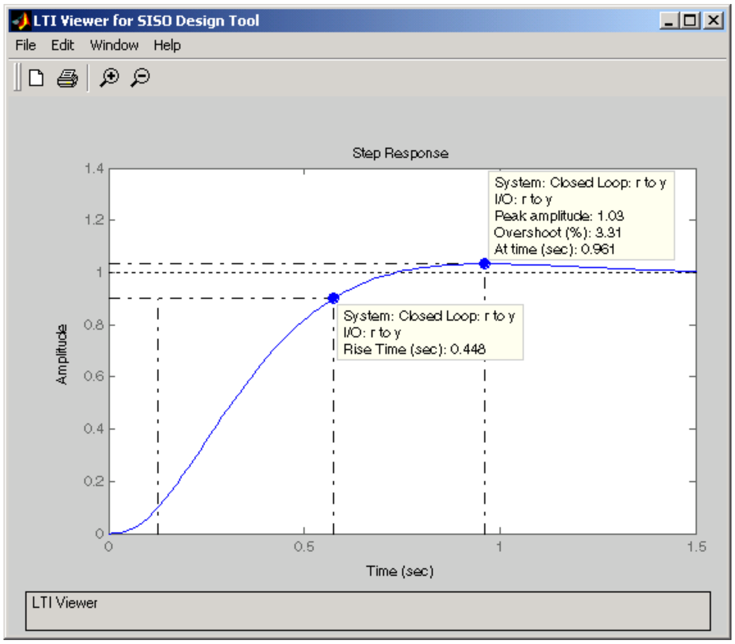
For this design, move the lead network pole to -28 and the zero to -4.3. The zero should be almost on top of the rightmost pole of the plant (the DC motor model). Adjust the compensator gain to 84. This gives the final design.


SISO Design for System sys_dc
File Edit View Compensators Analysis Tools Window Help
Current Compensator
C(s) = 84 x (1 + 0.23s) / s (1 + 0.036s)
F C G H
+/- FS
Open-Loop Bode Editor (C)
G.M.: 21.9 dB
Freq: 16.4 rad/sec
Stable loop
P.M.: 65.7 deg
Freq: 2.93 rad/sec
Frequency (rad/sec)
Open-Loop Nichols Editor (C)
G.M.: 21.9 dB @ 16.4 rad/sec
P.M.: 65.7 deg @ 2.93 rad/sec
Stable loop
Open-Loop Phase (deg)

The open-loop Bode plot is now visible.

Final Nichols Plot Design for the DC Motor Compensator

The gain and phase margins are 21.9 dB and 65.7 degrees, respectively. Inspect the closed-loop step response to see if the rise time and overshoot requirements are met.



System: Closed Loop: r to y
I/O: r to y
Peak amplitude: 1.03
Overshoot (%): 3.31
At time (sec): 0.961

System: Closed Loop: r to y
I/O: r to y
Rise Time (sec): 0.448

Closed-Loop Step Response for the Final Compensator Design

As this figure shows, the rise time is 0.448 second, and the overshoot is a little over 3%. This satisfies the rest of the design requirements.

Functions for Compensator Design

The term control system *design* refers to the process of selecting feedback gains that meet design specifications in a closed-loop control system. Most design methods are iterative, combining parameter selection with analysis, simulation, and insight into the dynamics of the plant. In addition to the SISO Design Tool, the Control System Toolbox provides a set of commands that you can use for a broader range of control applications, including

- Classical SISO design
- Modern MIMO design techniques such as pole placement and linear quadratic Gaussian (LQG) methods

Root Locus Design

The following table summarizes the commands for designing compensators using root locus design techniques.

Function	**Description**
`pzmap`	Pole-zero map
`rlocus`	Evans root locus plot
`sgrid`	Continuous ω_n, ζ grid for root locus plots
`sisotool`	Root Locus Design GUI
`zgrid`	Discrete ω_n, ζ grid for root locus plots

Pole Placement

The closed-loop pole locations have a direct impact on time response characteristics such as rise time, settling time, and transient oscillations. Root locus uses compensator gains to move closed-loop poles to achieve design specifications for SISO systems. You can, however, use state-space techniques to assign closed-loop poles. This design technique is known as *pole placement*, which differs from root locus in the following ways:

- Using pole placement techniques, you can design dynamic compensators.
- Pole placement techniques are applicable to MIMO systems.

Pole placement requires a state-space model of the system (use `ss` to convert other model formats to state space). In continuous time, such models are of the form

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$$

where u is the vector of control inputs, x is the state vector, and y is the vector of measurements.

State-Feedback Gain Selection

Under state feedback $u = -Kx$, the closed-loop dynamics are given by

$$\dot{x} = (A - BK)\,x$$

and the closed-loop poles are the eigenvalues of $A - BK$. Using the `place` command, you can compute a gain matrix K that assigns these poles to any desired locations in the complex plane (provided that (A, B) is controllable).

For example, for state matrices `A` and `B`, and vector `p` that contains the desired locations of the closed loop poles,

```
K = place(A,B,p);
```

computes an appropriate gain matrix `K`.

State Estimator Design

You cannot implement the state-feedback law $u = -Kx$ unless the full state x is measured. However, you can construct a state estimate ξ such that the law $u = -K\xi$ retains similar pole assignment and closed-loop properties. You can achieve this by designing a state estimator (or observer) of the form

$$\dot{\xi} = A\xi + Bu + L(y - C\xi - Du)$$

The estimator poles are the eigenvalues of $A - LC$, which can be arbitrarily assigned by proper selection of the estimator gain matrix L, provided that (C, A) is observable. Generally, the estimator dynamics should be faster than the controller dynamics (eigenvalues of $A - BK$).

Use the `place` command to calculate the L matrix

```
L = place(A',C',q)
```

where `A` and `C` are the state and output matrices, and `q` is the vector containing the desired closed-loop poles for the observer.

Replacing x by its estimate ξ in $u = -Kx$ yields the dynamic output-feedback compensator

$$\dot{\xi} = [A - LC - (B - LD)K]\xi + Ly$$
$$u = -K\xi$$

Note that the resulting closed-loop dynamics are

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}, \qquad e = x - \xi$$

Hence, you actually assign all closed-loop poles by independently placing the eigenvalues of $A - BK$ and $A - LC$.

Example. Given a continuous-time state-space model

```
sys_pp = ss(A,B,C,D)
```

with seven outputs and four inputs, suppose you have designed

- A state-feedback controller gain `K` using inputs 1, 2, and 4 of the plant as control inputs
- A state estimator with gain `L` using outputs 4, 7, and 1 of the plant as sensors
- Input 3 of the plant as an additional known input

You can then connect the controller and estimator and form the dynamic compensator using this code.

```
controls = [1,2,4];
sensors = [4,7,1];
known = [3];
regulator = reg(sys_pp,K,L,sensors,known,controls)
```

Pole Placement Tools

The Control System Toolbox contains functions to

- Compute gain matrices K and L that achieve the desired closed-loop pole locations.
- Form the state estimator and dynamic compensator using these gains.

The following table summarizes the commands for pole placement.

Command	**Description**
`acker`	SISO pole placement
`estim`	Form state estimator given estimator gain
`place`	MIMO pole placement
`reg`	Form output-feedback compensator given state-feedback and estimator gains

The function `acker` is limited to SISO systems and should only be used for systems with a small number of states. The function `place` is a more general and numerically robust alternative to `acker`.

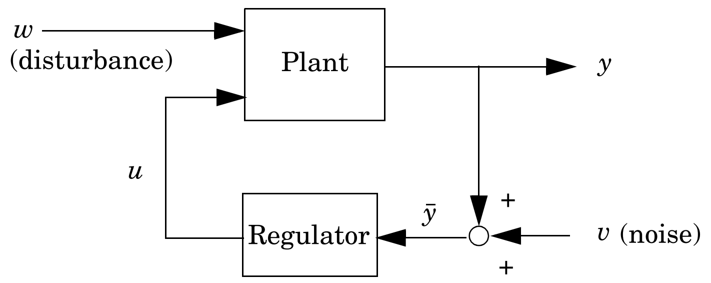
Caution. Pole placement can be badly conditioned if you choose unrealistic pole locations. In particular, you should avoid

- Placing multiple poles at the same location.
- Moving poles that are weakly controllable or observable. This typically requires high gain, which in turn makes the entire closed-loop eigenstructure very sensitive to perturbations.

Linear-Quadratic-Gaussian (LQG) Design

Linear-quadratic-Gaussian (LQG) control is a modern state-space technique for designing optimal dynamic regulators. It enables you to trade off regulation performance and control effort, and to take into account process disturbances and measurement noise. Like pole placement, LQG design requires a state-space model of the plant (use `ss` to convert other model formats to state space). This section focuses on the continuous-time case (see the reference pages for `dlqr` and `kalman` for details on discrete-time LQG design).

LQG design addresses the following regulation problem.



The goal is to regulate the output y around zero. The plant is subject to disturbances w and is driven by controls u. The regulator relies on the noisy measurements $\bar{y} = y + v$ to generate these controls. The plant state and measurement equations are of the form

$$\dot{x} = Ax + Bu + Gw$$

$$\bar{y} = Cx + Du + Hw + v$$

and both w and v are modeled as white noise.

The LQG regulator consists of an optimal state-feedback gain and a Kalman state estimator. You can design these two components independently as shown next.

Optimal State-Feedback Gain

In LQG control, the regulation performance is measured by a quadratic performance criterion of the form

$$J(u) = \int_0^{\infty} \{x^T Qx + 2x^T Nu + u^T Ru\}\ dt$$

The weighting matrices Q, N, and R are user specified and define the trade-off between regulation performance (how fast $x(t)$ goes to zero) and control effort.

The first design step seeks a state-feedback law $u = -Kx$ that minimizes the cost function $J(u)$. The minimizing gain matrix K is obtained by solving an algebraic Riccati equation. This gain is called the *LQ-optimal* gain.

Syntax. Given the (A,B,C,D) matrices of the system, and the weighting matrices Q, R, and N, use the lqr command to construct the LQ-optimal gain, K.

```
K= lqr(A,B,Q,R,N)
```

If N is omitted, by default its value is 0.

Kalman State Estimator

As in the case of pole placement, the LQ-optimal state feedback $u = -Kx$ is not implementable without full state measurement. It is possible, however, to derive a state estimate such that $u = -K\hat{x}$ remains optimal for the output-feedback problem. This state estimate is generated by the Kalman filter

$$\frac{d}{dt}\hat{x} = A\hat{x} + Bu + L(\bar{y} - C\hat{x} - Du)$$

with inputs u (controls) and $\bar{y}$ (measurements). The noise covariance data

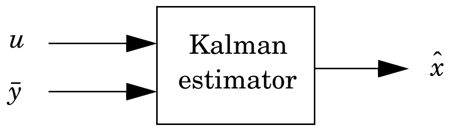
$$E(ww^T) = Q_n, \qquad E(vv^T) = R_n, \qquad E(wv^T) = N_n$$

determines the Kalman gain L through an algebraic Riccati equation.

The Kalman filter is an optimal estimator when dealing with Gaussian white noise. Specifically, it minimizes the asymptotic covariance

$$\lim_{t \to \infty} E((x - \hat{x})(x - \hat{x})^T)$$

of the estimation error $x - \hat{x}$.



Syntax. Use the kalman function to construct a Kalman filter.

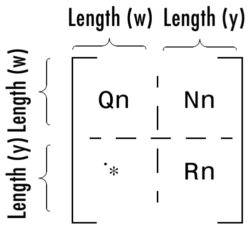
```
[kest,L,P] = kalman(sys_kf,Qn,Rn,Nn);
```

returns a state-space model kest of the Kalman estimator given the plant model sys_kf and the noise covariance data, Qn, Rn, and Nn. The plant model equations are the following.

$$\begin{aligned}\dot{x} &= Ax + Bu + Gw \\ \bar{y} &= Cx + Du + Hw + v\end{aligned}$$

where w and v are modeled as white noise. `L` is the Kalman gain and `P` the covariance matrix.

The figure below shows the required dimensions for `Qn`, `Rn`, and `Nn`. If `Nn` is 0, you can omit it.

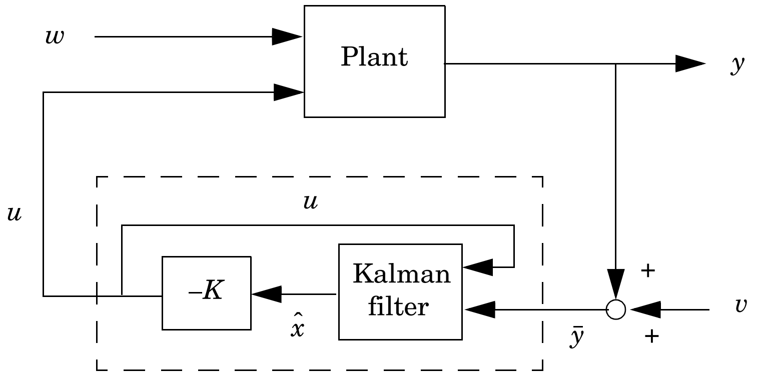


Required Dimensions for Qn, Rn, and Nn

For a complete example of a Kalman filter implementation, see “Kalman Filtering” under “Design Case Studies” in the Control System Toolbox online documentation.

LQG Regulator

To form the LQG regulator, simply connect the Kalman filter and LQ-optimal gain K as shown below.



LQG regulator

This regulator has state-space equations

$$\frac{d}{dt}\hat{x} = \left[A - LC - (B - LD)K\right]\hat{x} + L\bar{y}$$
$$u = -K\hat{x}$$

Syntax. Assuming you have constructed the Kalman filter, `kest`, and the compensator, `K`, use the `lqgreg` command to create the LQG regulator.

```
regulator = lqgreg(kest, K);
```

See the example LQG Regulation under Design Case Studies online for a more complete discussion of construction LQG regulators.

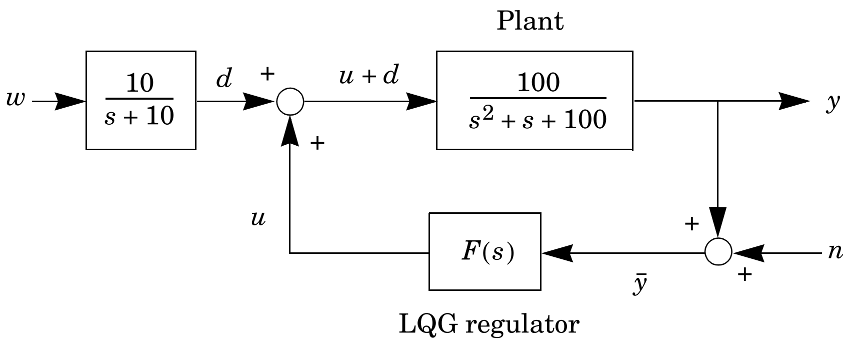
LQG Design Tools

The Control System Toolbox contains functions to perform the three LQG design steps outlined above. These functions cover both continuous and discrete problems as well as the design of discrete LQG regulators for continuous plants. The following table summarizes the LQG design commands.

Command	**Description**
`care`	Solve continuous-time algebraic Riccati equations
`dare`	Solve discrete-time algebraic Riccati equations
`dlqr`	LQ-optimal gain for discrete systems
`kalman`	Kalman estimator
`kalmd`	Discrete Kalman estimator for continuous plant
`lqgreg`	Form LQG regulator given LQ gain and Kalman filter
`lqr`	LQ-optimal gain for continuous systems
`lqrd`	Discrete LQ gain for continuous plant
`lqry`	LQ-optimal gain with output weighting

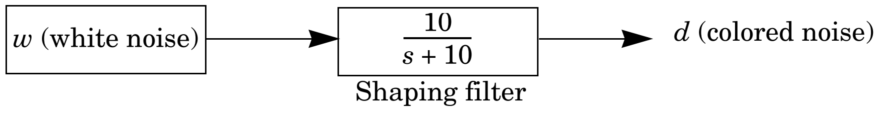
Example: LQG Design

As an example of LQG design, consider the regulation problem illustrated below.

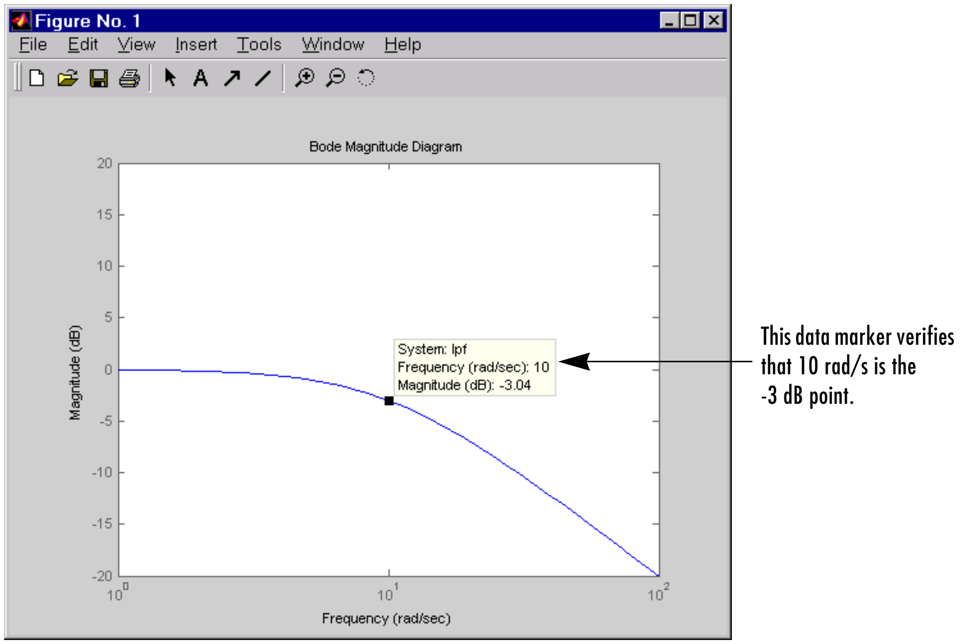


Simple Regulation Loop

The goal is to regulate the plant output y around zero. The input disturbance d is low frequency with power spectral density (PSD) concentrated below 10 rad/s. For LQG design purposes, it is modeled as white noise driving a lowpass filter with a cutoff at 10 rad/s, as this figure shows.



This figure shows the Bode magnitude of the shaping filter.



Bode Magnitude of the Lowpass Filter

There is some measurement noise n , with noise intensity given by

$$E(n^2) = 0.01$$

Use the cost function

$$J(u) = \int_0^\infty (10y^2 + u^2)dt$$

to specify the trade-off between regulation performance and cost of control. Note that an open-loop state-space model is

$$\begin{aligned} \dot{x} &= Ax + Bu + Bd && \text{(state equations)} \\ \bar{y} &= Cx + n && \text{(measurements)} \end{aligned}$$

where (A, B, C) is a state-space realization of $100/(s^2 + s + 100)$.

The following commands design the optimal LQG regulator $F(s)$ for this problem.

```
sys = ss(tf(100,[1 1 100])) % State-space plant model

% Design LQ-optimal gain K
K = lqry(sys,10,1) % u =  Kx minimizes J(u)

% Separate control input u and disturbance input d
P = sys(:,[1 1]);
% input [u;d], output y

% Design Kalman state estimator Kest.
Kest = kalman(P,1,0.01)

% Form LQG regulator = LQ gain + Kalman filter.
F = lqgreg(Kest,K)
```

The last command returns a state-space model `F` of the LQG regulator $F(s)$. Note that `lqry`, `kalman`, and `lqgreg` perform discrete-time LQG design when applied to discrete plants.

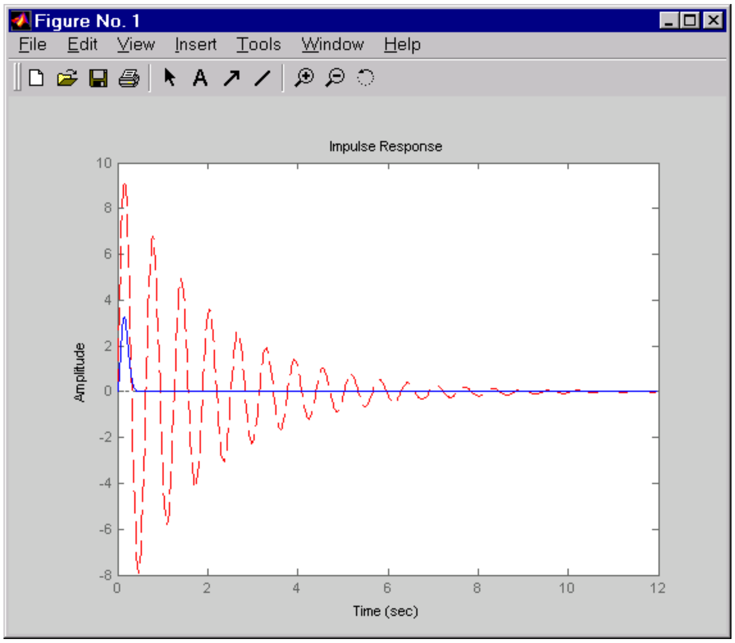
To validate the design, close the loop with `feedback`, create and add the lowpass filter in series with the closed-loop system, and compare the open- and closed-loop impulse responses by using the `impulse` function.

```
% Close loop
clsys = feedback(sys,F,+1)   % Note positive feedback.

% Create the lowpass filter and add it in series with clsys.
s = tf('s');
lpf= 10/(s+10) ;
clsys_fin = lpf*clsys;

% Open- vs. closed-loop impulse responses
impulse(sys,'r--',clsys_fin,'b-')
```

This figure compares the open- and closed-loop impulse responses for this example.



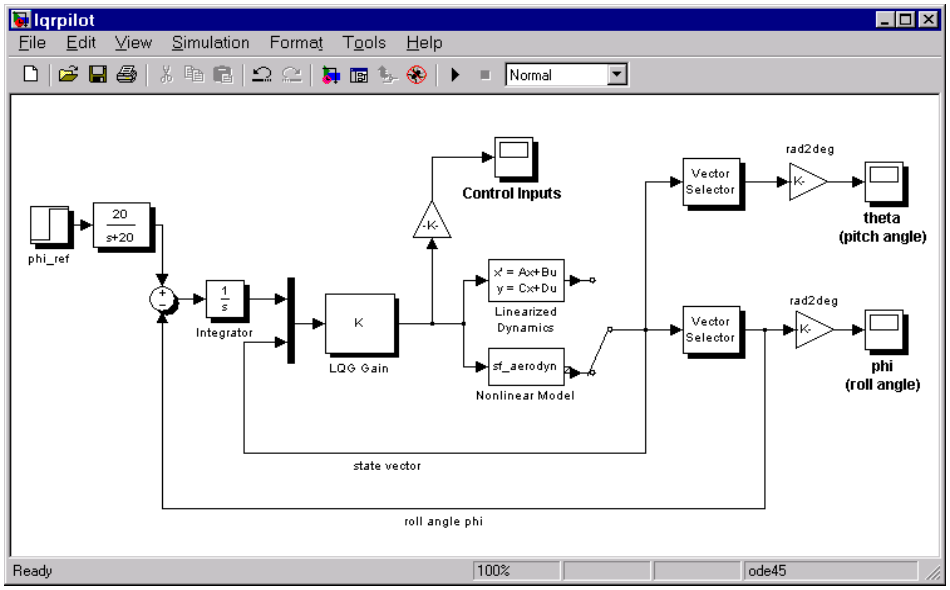
Comparison of Open- and Closed-Loop Impulse Response for the LQG Example

Example: LQG Design for Set Point Tracking

The standard LQG problem is to regulate the plant output around zero. The previous section, "Example: LQG Design," describes the classical LQG regulation problem.

You can also apply the LQG design technique to tracking problems, where the goal is to track a reference input (or set point) to the system. To recast the regulator as a tracking problem, you must compare the output y to the reference signal. The goal is then to drive the error between the output and the reference to zero. A common practice is to add an integrator to the error signal, $e = y - r$, to drive it to zero.

This Simulink block diagram shows a tracking problem in aircraft autopilot design. To open this diagram, type `lqrpilot` at the MATLAB prompt.



A Tracking Loop

Key features of this diagram to note are the following:

- The Linearized Dynamics block contains the linearized airframe.
- `sf_aerodyn` is an S-Function block that contains the nonlinear equations for $(\theta,\phi) = (0,15°)$
- The error signal between ϕ and the ϕ_{ref} is passed through an integrator. This aids in driving the error to zero.

State-Space Equations for an Airframe

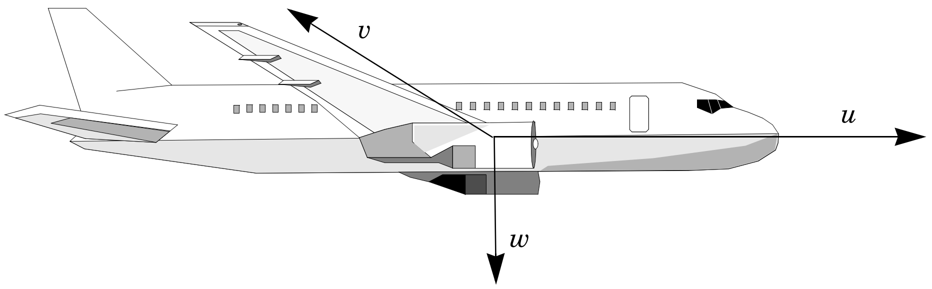
Beginning with the standard state-space equation

$$\dot{x} = Ax + Bu$$

where

$$x = [u, v, w, p, q, r, \theta, \phi]^T$$

The variables u, v, and w are the three velocities with respect to the body frame, which is shown in the figure below.



A Body Coordinate Frame for an Aircraft

The variables θ and ϕ are roll and pitch, and p, q, and r are the roll, pitch, and yaw rates, respectively.

The airframe dynamics are nonlinear. The equation below shows the nonlinear components added to the state-space equation.

$$\dot{x} = Ax + Bu + \begin{bmatrix} -g\sin\theta \\ g\cos\theta\sin\phi \\ g\cos\theta\cos\phi \\ 0 \\ 0 \\ 0 \\ 0 \\ q\cos\phi - r\sin\phi \\ (q\sin\phi + r\cos\phi)\cdot\tan\theta \end{bmatrix}$$

Nonlinear Component of the State-Space Equation

To see the numerical values for A and B, type

```
load lqrpilot
A, B
```

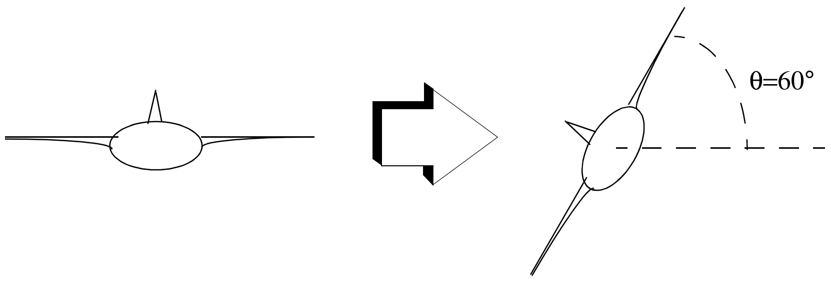
at the MATLAB prompt.

Trimming

For LQG design purposes, the nonlinear dynamics are trimmed at $\phi = 15°$ and p, q, r, and θ set to zero. Since u, v, and w do not enter into the nonlinear term in the preceding figure, this amounts to linearizing around $(\phi, \theta) = (0, 15°)$ with all remaining states set to zero. The resulting state matrix of the linearized model is called `A15`.

Problem Definition

The goal to perform a steady coordinated turn, as shown in this figure.



An Aircraft Making a 60° Turn

To achieve this goal, you must design a controller that commands a steady turn by going through a 60° roll. In addition, assume that θ, the pitch angle, is required to stay as close to zero as possible.

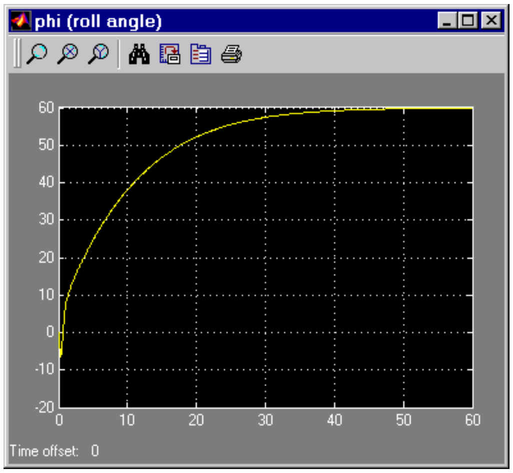
Results

To calculate the LQG gain matrix, `K`, type

```
lqrdes
```

at the MATLAB prompt. Then start the `lqrpilot` model with the nonlinear model, `sf_aerodyn`, selected.

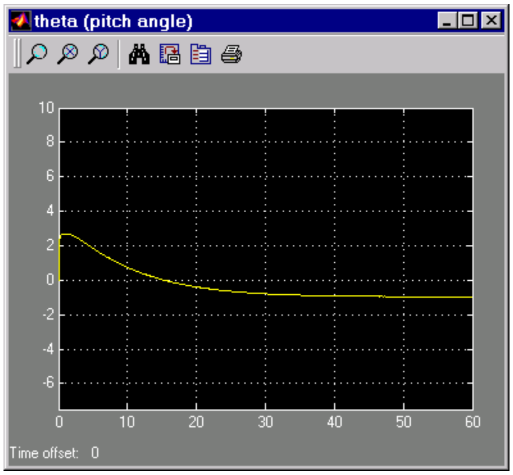
This figure shows the response of ϕ to the 60° step command.



Tracking the Roll Step Command

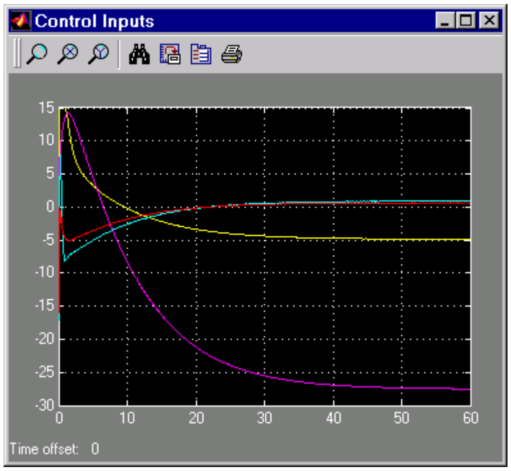
As you can see, the system tracks the commanded 60° roll in about 60 seconds.

Another goal was to keep θ , the pitch angle, relatively small. This figure shows how well the LQG controller did.



Minimizing the Displacement in the Pitch Angle, Theta

Finally, this figure shows the control inputs.



The Control Inputs for the LQG Tracking Problem

Try adjusting the `Q` and `R` matrices in `lqrdes.m` and inspecting the control inputs and the system states, making sure to rerun `lqrdes` to update the LQG gain matrix `K`. Through trial and error, you may improve the response time of this design. Also, compare the linear and nonlinear designs to see the effects of the nonlinearities on the system performance.

Learning More

Demos

To see more Control System Toolbox examples, type

```
demo
```

at the MATLAB prompt. This opens the **Demo** pane in the Help browser. Select **Control System Toolbox** under **Toolboxes** to list the available demos.

Alternatively, if you have the Help browser open, you can access the **Demo** pane directly and follow the same procedure.

Online Help

For a more detailed explanation of any of the topics covered in this book, see the documentation listed under Control System Toolbox in the Help Navigator.

The Help Navigator supports string searches. You can specify strings and the online manuals that you want to search. To begin a search, click the **Search** tab. There is also an index available; click the **Index** tab to view it.

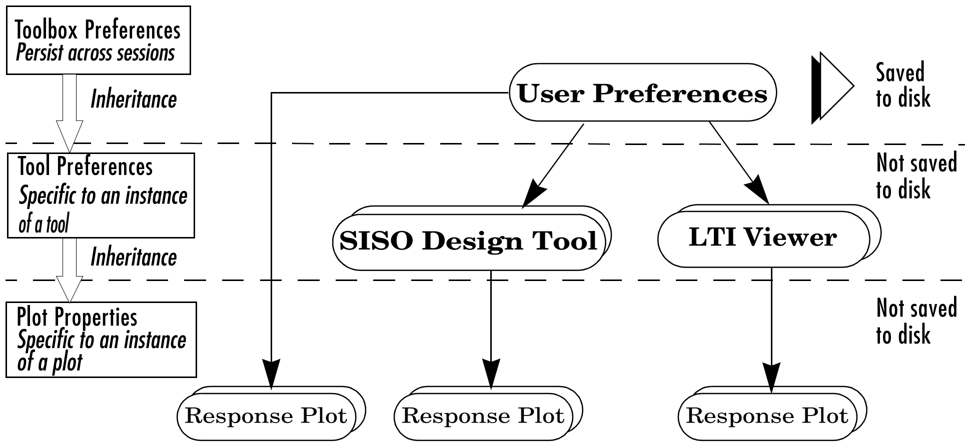
Setting Plot Preferences and Properties

The Control System Toolbox provides three graphical user interfaces (GUIs) that give you control over the visualization of time and frequency plots generated by the toolbox:

- Toolbox Preferences
- Tool Preferences
- Plot Properties

Preferences refer to global options that you can save from session to session or to any LTI Viewer or SISO Design Tool that you open during a single session. *Properties* are options that apply only to the current window. This section gives an overview of the three GUIs; see the online help system for complete descriptions.

Although you can set plot properties in any response plot, you can use the Toolbox Preferences Editor to set properties for any response plot the Control System Toolbox generates. This figure shows the inheritance hierarchy from toolbox preference to plot properties.



Preference and Property Inheritance Hierarchy

You can activate preference and plot editors by doing the following:

- Toolbox preferences — Select **Toolbox Preferences** under **File** in either the LTI Viewer or the SISO Design Tool.
- Tool preferences — Select **SISO Tool Preferences** under **Edit** for the SISO Design Tool and **Viewer Preferences** under **Edit** in the LTI Viewer.
- Plot properties — Double-click any response plot created by the Control System Toolbox or select **Properties** from the right-click menus.

For a complete discussion of how to use property and preference editors, see “Customization” in the online help under “Control System Toolbox.”

The MathWorks Online

For the very latest information about the Control System Toolbox and other MathWorks products, point your Web browser to

```
www.mathworks.com
```

and use your Internet news reader to access the newsgroup

```
comp.soft-sys.matlab
```

Many books that use MATLAB and the Control System Toolbox to explain control engineering concepts are available from different publishers. A booklet titled MATLAB Based Books is available from The MathWorks and an up-to-date list is available on the Web site.

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